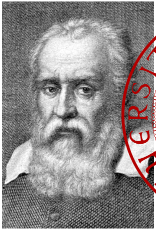
Stochastic control of HVAC systems: a learning-based approach

Damiano Varagnolo

Something about me



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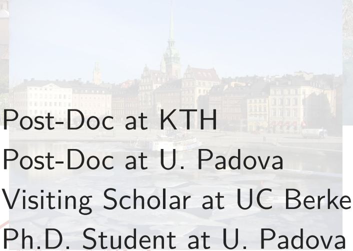
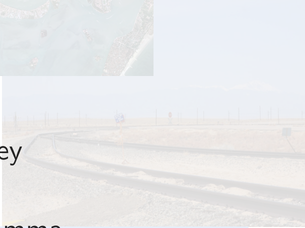



Post-Doc at KTH

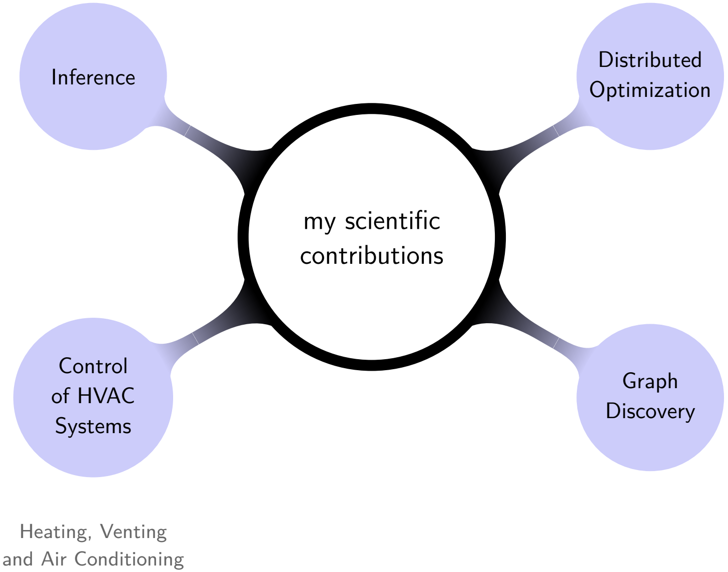
Post-Doc at U. Padova

Visiting Scholar at UC Berkeley

Ph.D. Student at U. Padova

Software Engineer at Tecnogamma

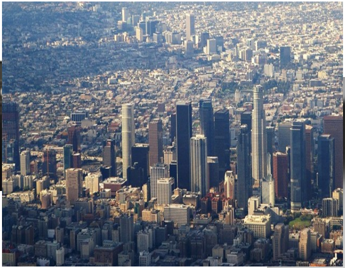
MS. Student at U. Padova


Inference
Distributed Optimization
my scientific contributions
Control of HVAC Systems
Graph Discovery
Heating, Venting and Air Conditioning

Motivations

Motivations

reduce energy consumption



Motivations

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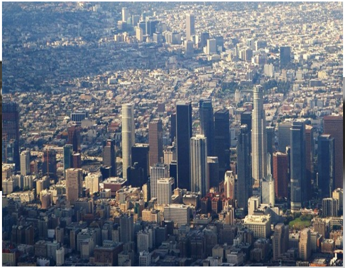

maintain quality indexes

Motivations

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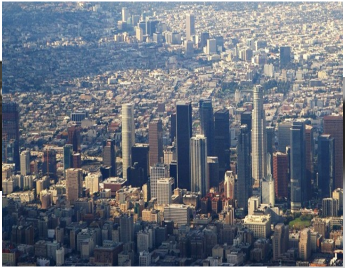
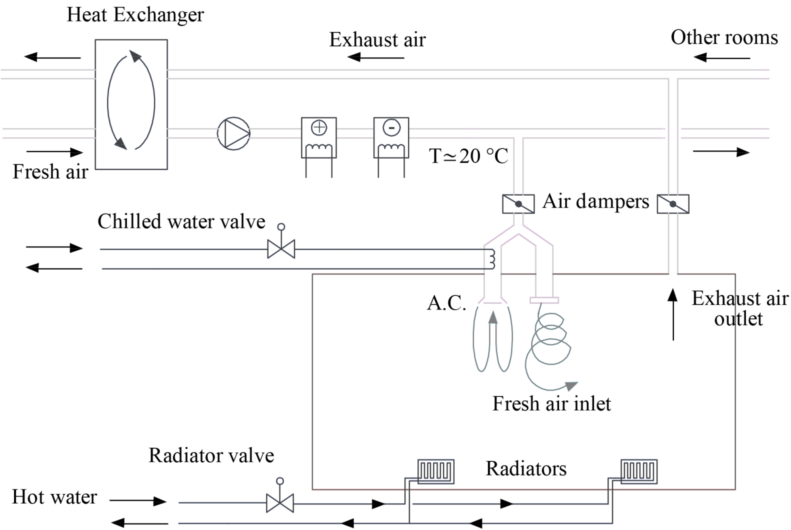


maintain quality indexes

space conditioning: 10 – 20 % of global final energy consumption

A typical HVAC system



General methodology

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☞ **use buildings thermal capacity** ☜

General methodology

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use accurate models

use forecasts

General methodology

☞ **use buildings thermal capacity** ☜

use accurate models

use forecasts

use Model Predictive Control (MPC)

Example

Model (u inputs, w disturbances)

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + Ew(k) \\ y(k) &= Cx(k) \end{aligned}$$

Predicted evolution (given u and w)

$$\boldsymbol{Y} = \boldsymbol{C}(\boldsymbol{A}x_0 + \boldsymbol{B}\boldsymbol{U} + \boldsymbol{E}\boldsymbol{W})$$

Model Predictive Control (MPC)

$$\begin{aligned} \min_{\boldsymbol{U}} \quad & c^T \boldsymbol{U} \\ \text{s.t.} \quad & \boldsymbol{C}(\boldsymbol{A}x_0 + \boldsymbol{B}\boldsymbol{U} + \boldsymbol{E}\boldsymbol{W}) \in \text{comfort bounds} \end{aligned}$$

Literature review

- Ma et al. (2012)
  Fast stochastic MPC with optimal risk allocation applied to building control systems
  Conference on Decision and Control
- Oldewurtel et al. (2012)
  Use of model predictive control and weather forecasts for energy efficient building climate control
  Energy and Buildings
- Salsbury et al. (2012)
  Predictive control methods to improve energy efficiency and reduce demand in buildings
  Computers and Chemical Engineering
- Mady et al. (2011)
  Stochastic model predictive controller for the integration of building use and temperature regulation
  Conference on Artificial Intelligence

Contributions

A. Ebadat, G. Bottegal, D. Varagnolo, B. Wahlberg, K.H. Johansson
Estimation of building occupancy levels through environmental signals deconvolution
ACM Workshop On Embedded Systems For Energy-Efficient Buildings, 2013

A. Parisio, D. Varagnolo, D. Risberg, G. Pattarello, M. Molinari, K.H. Johansson
Randomized Model Predictive Control for HVAC Systems
ACM Workshop On Embedded Systems For Energy-Efficient Buildings, 2013

A. Parisio, M. Molinari, D. Varagnolo, K.H. Johansson
A Scenario-based Predictive Control Approach to Building HVAC Management Systems
IEEE Conference on Automation Science and Engineering, 2013

Contributions

robustness

Contributions

robustness

through ***learning*** the uncertainties

Contributions – Main Directions

Robust control of CO_2 and temperature

Estimation and modeling of occupancy levels

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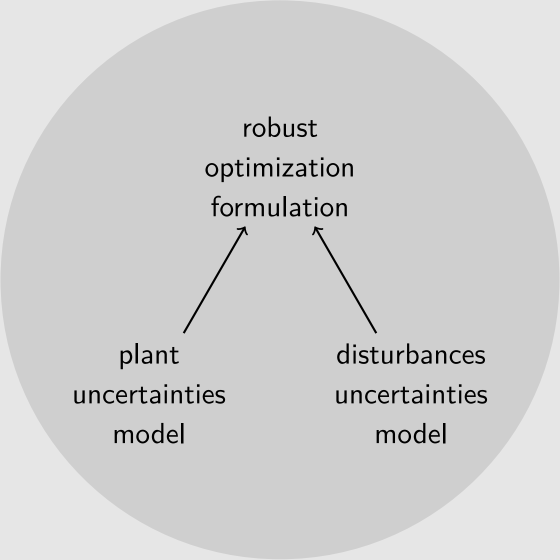
Roadmap


robust
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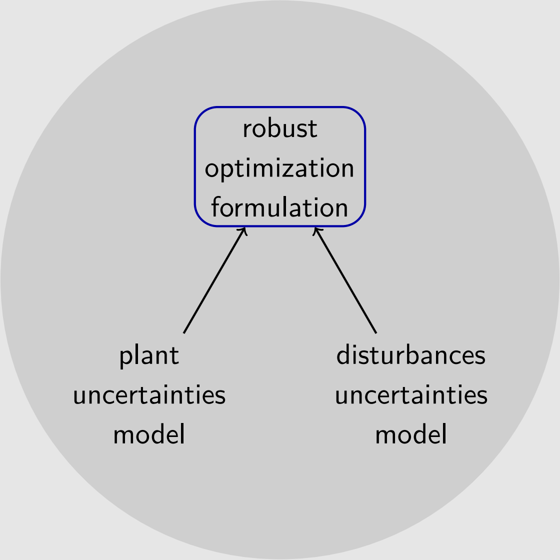
Roadmap


robust optimization formulation
plant uncertainties model
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Worst-case Robust Optimization

Problem *Ben-Tal, Nemirovsky, El Ghaoui, ...*

$$\begin{aligned} \min_{\theta \in \Theta} \quad & c^T \theta \\ \text{s.t.} \quad & f(\theta, \delta) \leq 0 \qquad \forall \delta \in \Delta \end{aligned} \tag{1}$$

where

- $\theta \in \Theta$, with Θ closed and convex
- $\delta \in \Delta$, with Δ generic
- $f(\theta, \delta)$ continuous and convex in θ for any fixed $\delta \in \Delta$

Worst-case Robust Optimization in HVAC systems

$$\boldsymbol{C}(\boldsymbol{A}x_0 + \boldsymbol{BU} + \boldsymbol{EW}) \in \text{comfort bounds} \qquad \forall \boldsymbol{W} \in \Delta$$

Question: how to compute Δ?

Worst-case Robust Optimization in HVAC systems

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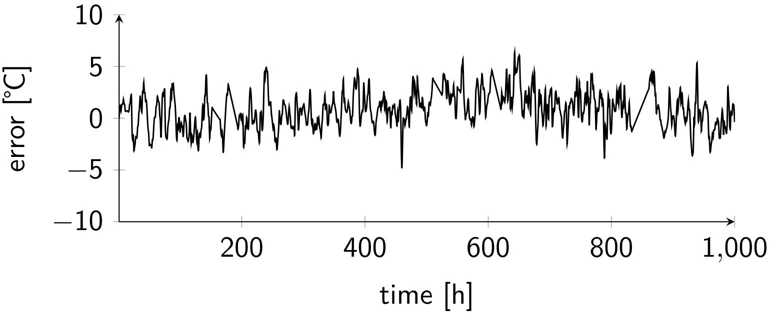
Question: how to compute Δ?

1 hour ahead forecast error – `www.wunderground.com`



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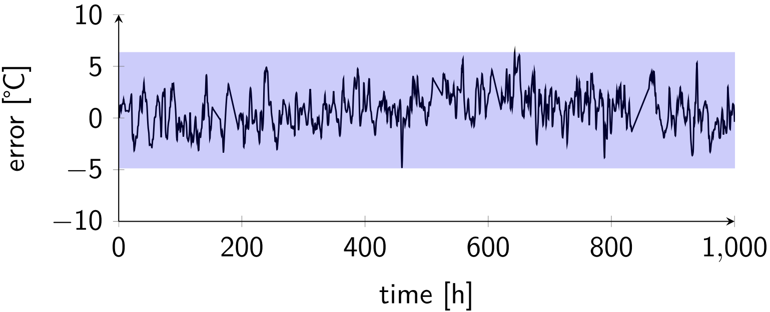

Chance-Constrained Robust Optimization

Problem *(Vajda, Prekopa, . . .)*

$$\begin{aligned} \min_{\theta \in \Theta} \quad & c^T \theta \\ \text{s.t.} \quad & \mathbb{P}\left[\delta \in \Delta \text{ s.t. } f(\theta, \delta) \leq 0\right] \geq 1 - \alpha \end{aligned} \tag{2}$$

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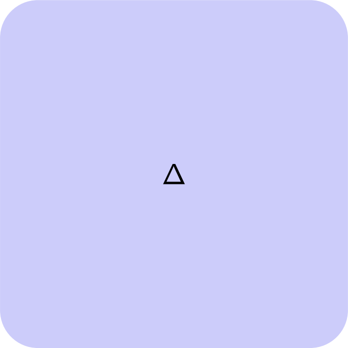

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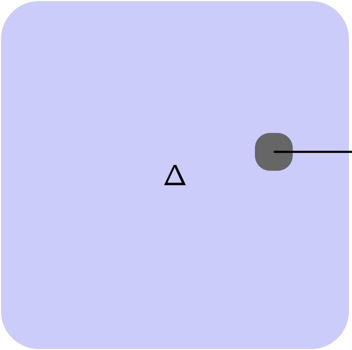

$$\left\{\delta \in \Delta \text{ s.t. } f(\boldsymbol{\theta_1}, \delta) > 0\right\}$$

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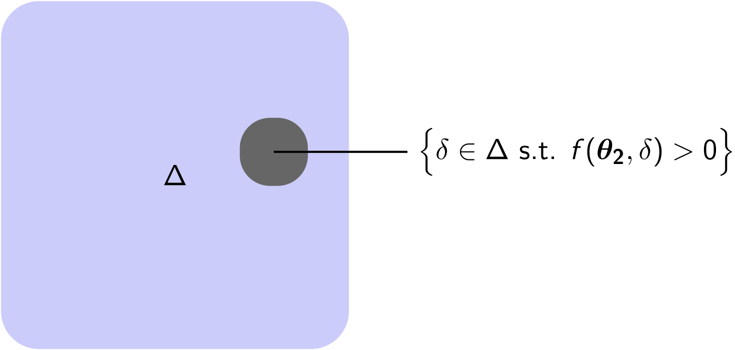

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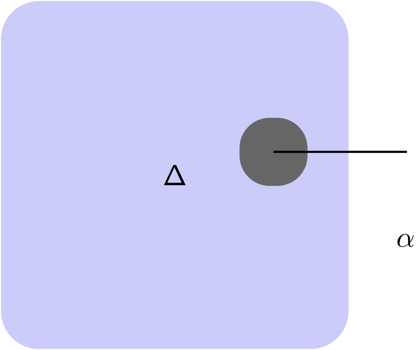


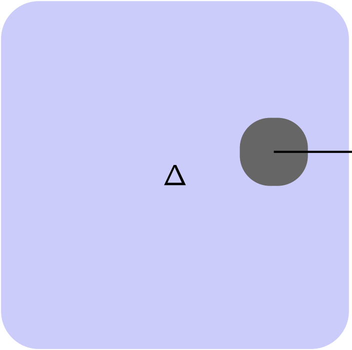
$\left\{\delta \in \Delta \text{ s.t. } f(\boldsymbol{\theta_2}, \delta) > 0\right\}$

α regulates the conservativity

Chance-Constrained Robust Optimization

Problem *(Vajda, Prekopa, ...)*

$$\begin{aligned} \min_{\theta\in\Theta} \quad & c^T\theta \\ \text{s.t.} \quad & \mathbb{P}\left[\delta \in \Delta \text{ s.t. } f(\theta,\delta) \le 0\right] \ge 1-\alpha \end{aligned} \tag{2}$$



$\left\{\delta \in \Delta \text{ s.t. } f(\boldsymbol{\theta_2},\delta) > 0\right\}$

α regulates the conservativity

may be NP-hard!

Scenario-Constrained Robust Optimization

Problem *(Campi, Calafiore, ...)*

$$\begin{aligned} \min_{\theta \in \Theta} \quad & c^T \theta \\ \text{s.t.} \quad & f(\theta, \delta^{(i)}) \leq 0 \qquad \forall i \in \{1, \ldots, N\} \end{aligned} \tag{3}$$

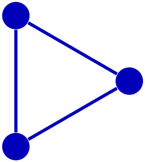
with:

- $\delta^{(i)}$'s = i.i.d. extractions of δ *(scenarios)*
- $N = \left\lceil \frac{2}{\alpha} \log \frac{1}{\beta} + 2n_\Theta + \frac{2n_\Theta}{\alpha} \log \frac{2}{\alpha} \right\rceil$
- $\alpha, \beta \in (0, 1)$ = confidence levels

(constraints may be substituted with $\max_i \left(f(\theta, \delta^{(i)}) \right) \leq 0$)

Worst-Case:

$$f(\theta, \delta) \leq 0 \quad \forall \delta \in \Delta$$



Scenario-Constrained:

$$f(\theta, \delta^{(i)}) \leq 0 \quad \forall i \in \{1, \ldots, N\}$$

Chance-Constrained:

$$\mathbb{P}\left[\delta \in \Delta \text{ s.t. } f(\theta, \delta) \leq 0\right] \geq 1 - \alpha$$

Comparisons Theorems *(Calafiore Campi 2006)*

- Scenario-Constrained infeasible $\Rightarrow$ Worst-Case infeasible
- Scenario-Constrained feasible $\Rightarrow$

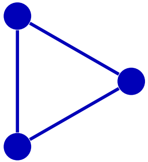
  - $c^T \theta^*_{SC} \leq c^T \theta^*_{WC}$
    (solution of Scenario-Constrained is not worse than Worst-Case)

  - $\mathbb{P}\left[c^T \theta^*_{CC}(\alpha) \leq c^T \theta^*_{SC}\right] \geq 1 - \beta$
  - $\mathbb{P}\left[c^T \theta^*_{SC} \leq c^T \theta^*_{CC}(\alpha')\right] \geq 1 - \beta$ with $\alpha' = \phi(\alpha, \beta) < \alpha$
    (w.h.p. solution of Scenario-Constrained is not better than Scenario-Constrained but also not too worse)

  - $\mathbb{P}\left[\quad \mathbb{P}\left[\delta \in \Delta \text{ s.t. } f(\theta^*_{SC}, \delta) \leq 0\right] \geq 1 - \alpha \quad\right] \geq 1 - \beta$
    (may happen that solution of Scenario-Constrained is not feasible for Scenario-Constrained)

Worst-Case:

$$f(\theta, \delta) \leq 0 \quad \forall \delta \in \Delta$$



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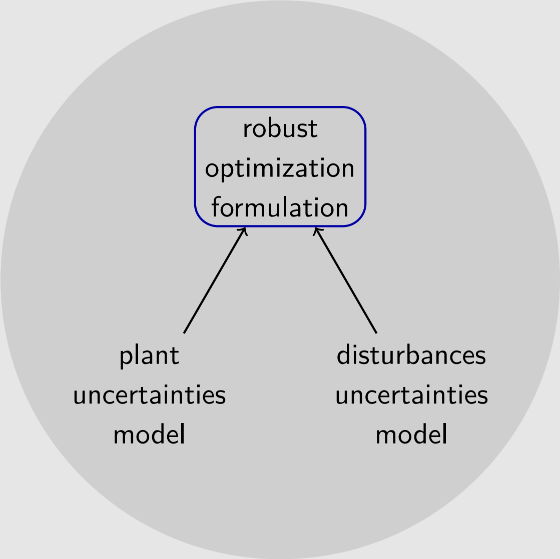
Roadmap


robust
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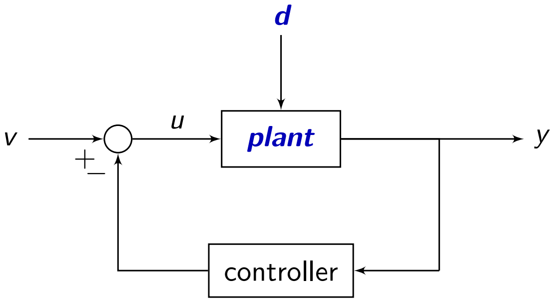
Sources of uncertainties



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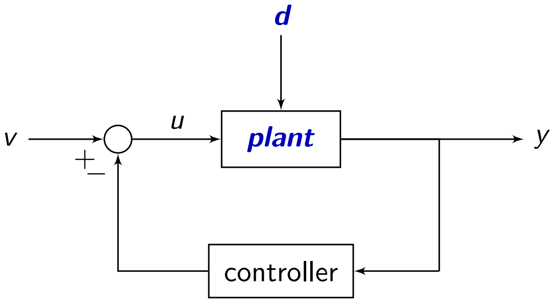
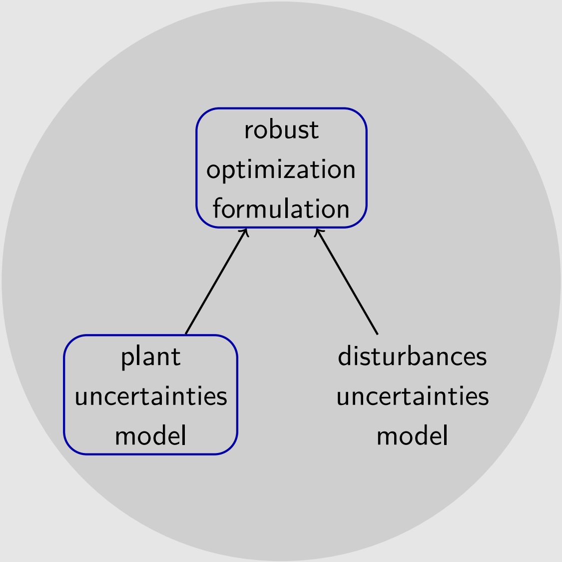


Our approach in deriving their distributions

plant : use nonparametric system identification tools

d : use copulas-based learning schemes

Roadmap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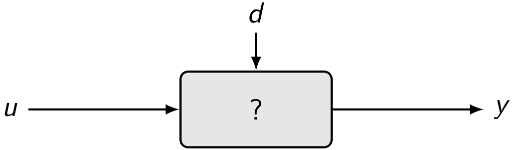
Nonparametric identification of LTIs



Nonparametric PEM approach:

$$\widehat{y}_{t|t-1} = \sum_{i=1}^{\infty} h'_i u_{t-i} + \sum_{i=1}^{\infty} h''_i y_{t-i}$$

System Identification = Regularized Function Estimation:

$$h^* = \arg\min_{h \in \bigstar} \sum_t \left(y_t - \widehat{y}_{t|t-1} \right)^2 + \gamma \, \|h\|^2_{\bigstar}$$

Nonparametric identification of LTIs

Theorem *(Pillonetto De Nicolao 2010)*

Let

$$K(x_1, x_2) = W\left(e^{-\beta x_1}, e^{-\beta x_2}\right)$$

$$W(s,t) = \begin{cases} \dfrac{s^2}{2}\left(t - \dfrac{s}{3}\right) & \text{if } s \leq t \\ \dfrac{t^2}{2}\left(s - \dfrac{t}{3}\right) & \text{if } s > t \end{cases}$$

If $h \sim \mathcal{GP}(0, K)$ then

$$\mathbb{P}\left[h = \text{imp. resp. of LTI BIBO stable system}\right] = 1$$

Nonparametric identification of LTIs

$$\min_{h \in H_K} \sum_t \left(y_t - \widehat{y}_{t|t-1} \right)^2 + \gamma \left\| h \right\|_K^2$$

returns:

- h^*, conditional expectation
- K^*, conditional autocovariance

Nonparametric identification of LTIs

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returns:

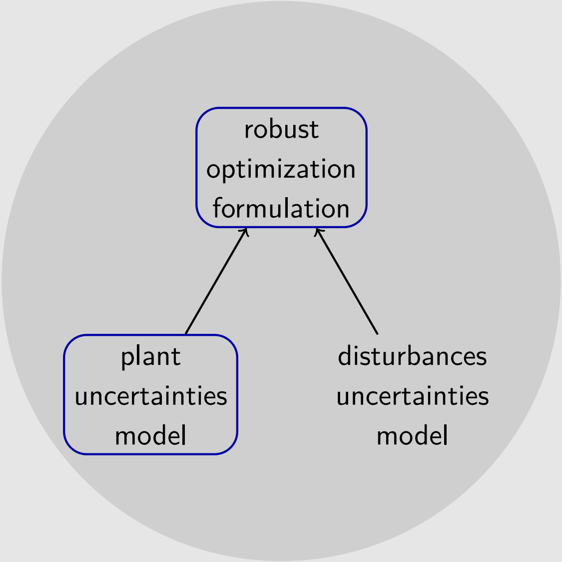
- h^*, conditional expectation
- K^*, conditional autocovariance

$\Rightarrow$ full probabilistic estimate:

$$\mathcal{GP}\left(h^*, K^*\right)$$

$\Rightarrow$ can extract i.i.d. samples $h^{(i)}$

Roadmap



robust optimization formulation
plant uncertainties model
disturbances uncertainties model

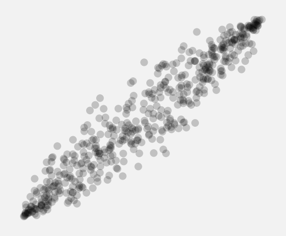
Problem definition

next step: learn the probability distribution of the disturbances

approach: use copulas-based learning techniques

Limitations of Gaussian Processes

Gaussian



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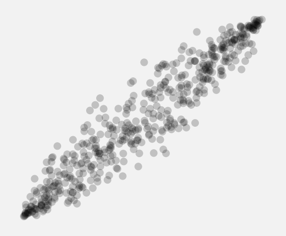
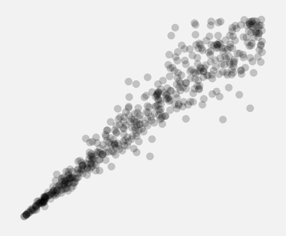


Clayton



Limitations of Gaussian Processes

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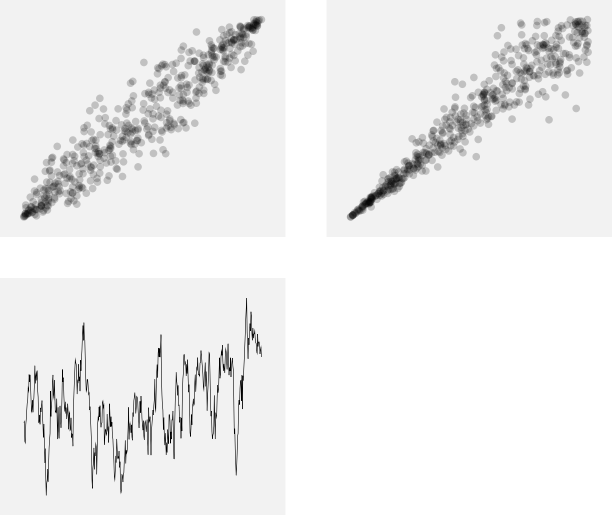

Limitations of Gaussian Processes

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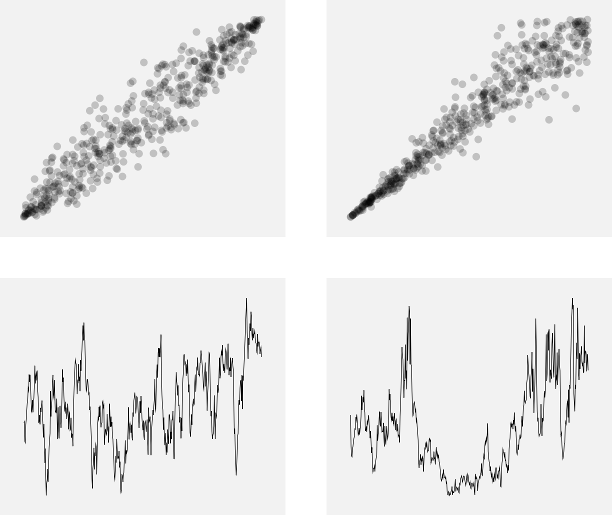

Describing Probabilities Using Copulas *(Sklar, Zimmer)*

$$\mathbb{F}_{\boldsymbol{w}}(a_1, \ldots, a_K) = \mathbb{C}\left(\mathbb{F}_{w_1}(a_1), \ldots, \mathbb{F}_{w_K}(a_K)\right) \qquad \mathbb{C} : [0,1]^K \mapsto [0,1]$$

In words, Joint distribution $=$ Copula $+$ Marginal distributions

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Pros

- completely generic
- separated modeling / learning of marginals / dependencies

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In words, Joint distribution = Copula + Marginal distributions

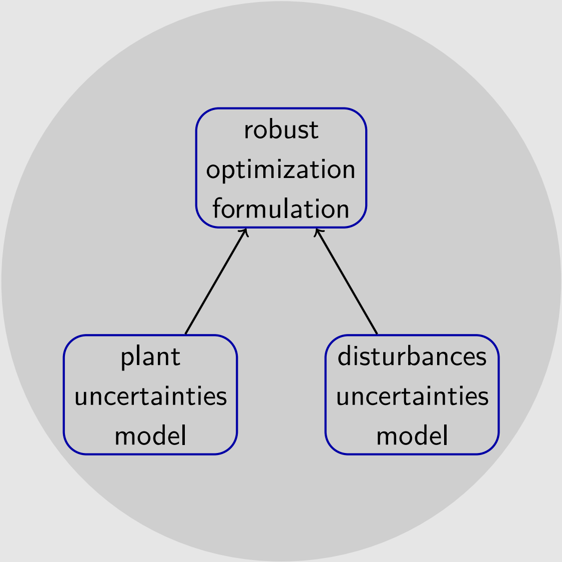
Pros

- completely generic
- separated modeling / learning of marginals / dependencies

Cons

- generating *scenarios* is computationally more expensive

Roadmap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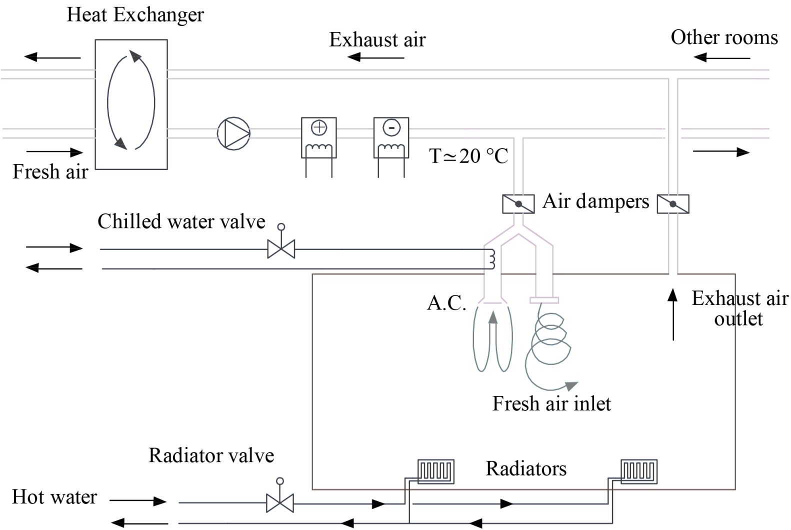
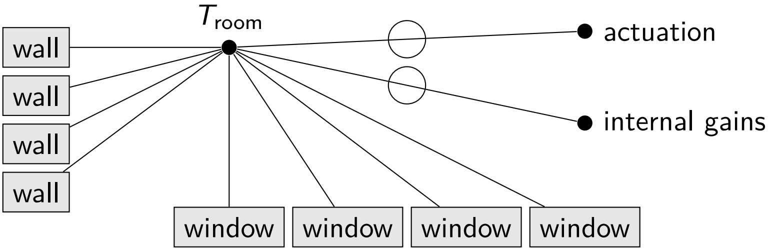
The controlled HVAC system

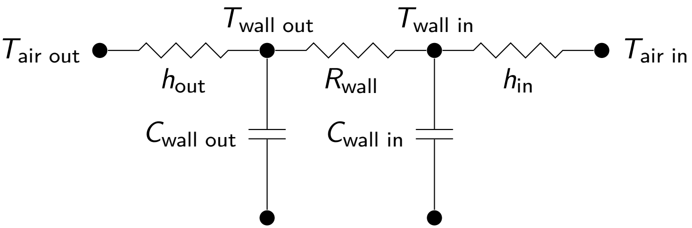


Necessity: model should be accurate and computationally tractable

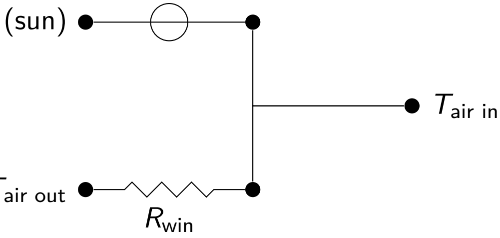
Our choice: RC-network (R $\leftrightarrow$ thermal resistance, C $\leftrightarrow$ thermal capacitance)



Wall model

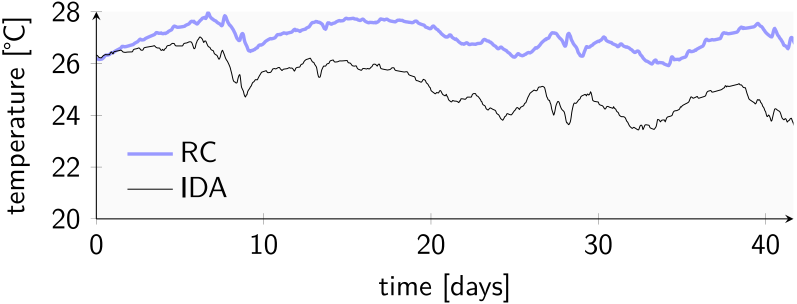


Window model



Building model

Validation against IDA-ICE



- simpler than commercial SW exploiting more complex libraries
- captures the most important buildings dynamics' characteristics

Scenario-based MPC

Thanks to the physics, separation of time scales

indoor air quality control → CO_2 control

indoor air quality control → thermal control

Scenario-based MPC

Thanks to the physics, separation of time scales

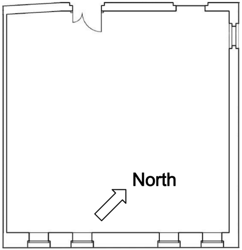
$$\text{indoor air quality control} \begin{cases} CO_2 \text{ control} \\ \text{thermal control} \end{cases}$$

Thanks to the linear models, linear programs

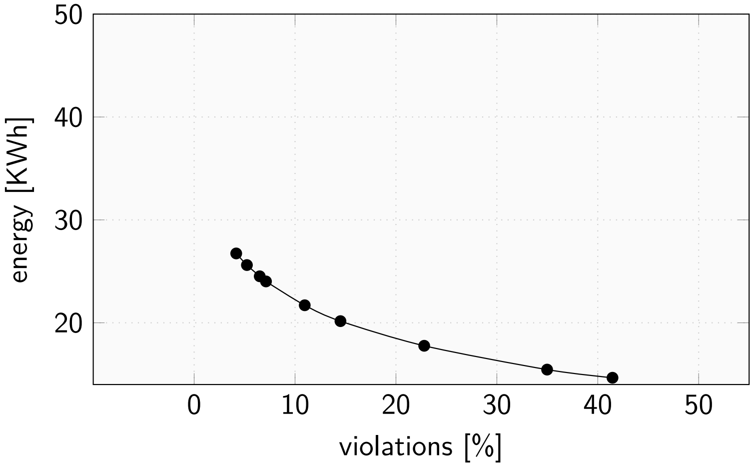
$$\begin{aligned} \min_{\boldsymbol{U}} \quad & c^T \boldsymbol{U} \\ \text{s.t.} \quad & \max_{i=1,\dots,N} \left(\boldsymbol{G}_u^{(i)} \boldsymbol{U} + \boldsymbol{G}_w^{(i)} \boldsymbol{W}^{(i)} - \boldsymbol{g}^{(i)} \right) \leq 0 \\ & \boldsymbol{F} \boldsymbol{U} \leq \boldsymbol{f} \end{aligned}$$

Numerical results

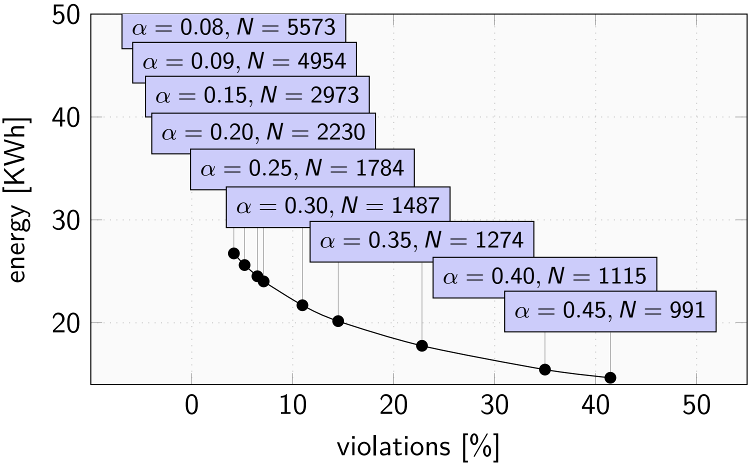
Room (`hvac.ee.kth.se`):



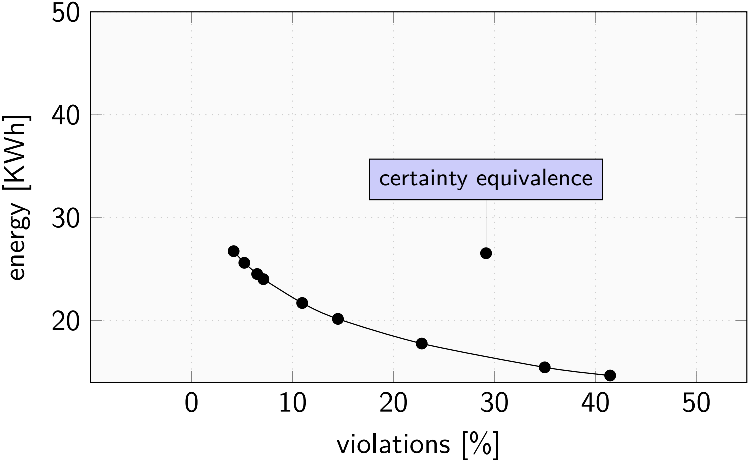
Numerical Results - Simulations



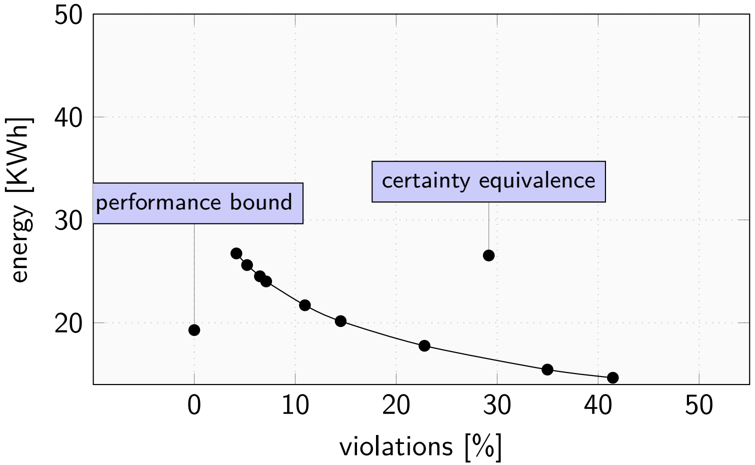
Numerical Results - Simulations

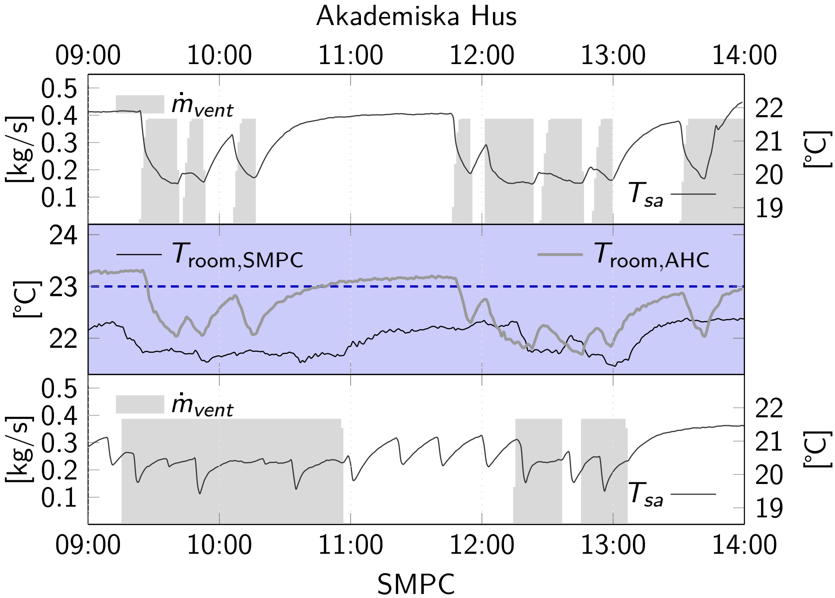


Numerical Results - Simulations



Numerical Results - Simulations




Akademiska Hus
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$T_{\text{room,SMPC}}$
$T_{\text{room,AHC}}$
0.5
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SMPC

Summary

aim: improve HVAC control through robustness

requires scenario-based control plus learning

results indicate noticeable savings

Directions

- learning from networks of buildings
- integration of smart-grid concepts
- global network of open HVAC testbeds

Stochastic control of HVAC systems: a learning-based approach

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entirely written in $\LaTeX 2_\varepsilon$ using Beamer and Tik Z