Analysis of Newton-Raphson Consensus for multi-agent convex optimization under asynchronous and lossy communications

Ruggero Carli Giuseppe Notarstefano
Luca Schenato **Damiano Varagnolo**

CDC 2015 - Osaka

December 14, 2015

Joint work with. . .

Ruggero Carli
Univ. of Padova

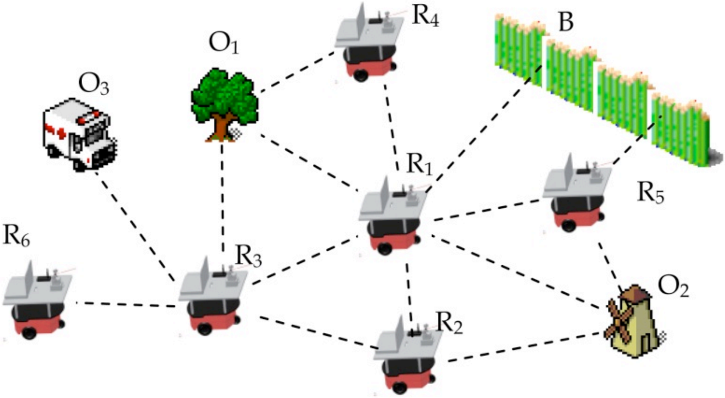
Giuseppe Notarstefano
Univ. of Lecce

Luca Schenato
Univ. of Padova

why distributed optimization?

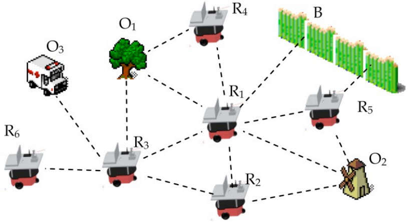
Example: distributed localization

Range-bearing measurements:



Example: distributed localization

Range-bearing measurements:

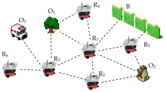


Definitions

- $x_i \in \mathbb{R}^2$ = position of robot i
- $\boldsymbol{x} = (x_1, \ldots, x_N) \in \mathbb{R}^{2N}$ = summary of all the positions
- $z_{ij} \in \mathbb{R}^2 = x_i - x_j + \textit{noise}$ = noisy measurement of the distance between i and j

Example: distributed localization

Range-bearing measurements:



Definitions

- $x_i \in \mathbb{R}^2$ = position of robot i
- $\boldsymbol{x} = (x_1, \dots, x_N) \in \mathbb{R}^{2N}$ = summary of all the positions
- $z_{ij} \in \mathbb{R}^2 = x_i - x_j +$ *noise* = noisy measurement of the distance between i and j

Problem

$$\boldsymbol{x}^* = \arg \min_{\widetilde{\boldsymbol{x}} \in \mathbb{R}^{2N}} \sum_i \sum_{j \in \mathcal{N}_i} \|\widetilde{x}_i - \widetilde{x}_j - z_{ij}\|^2$$

what are the challenges?

- asynchronous communications
- broadcast communications
- no channel feedback

State of the art

synchronous communications:

ADMM (Bertsekas 1997, Boyd 2010, He 2011, Deng 2011, Johansson 2008, Mota 2012,. . .)

State of the art

synchronous communications:

ADMM (Bertsekas 1997, Boyd 2010, He 2011, Deng 2011, Johansson 2008, Mota 2012,. . .)

asynchronous communications with perfect channel feedback:

ADMM with symmetric gossip (Wei Ozdaglar 2012, Jakovetic 2011, . . .)

State of the art

synchronous communications:

ADMM (Bertsekas 1997, Boyd 2010, He 2011, Deng 2011, Johansson 2008, Mota 2012,. . .)

asynchronous communications with perfect channel feedback:

ADMM with symmetric gossip (Wei Ozdaglar 2012, Jakovetic 2011, . . .)

asynchronous communications without perfect channel feedback:

Distributed quadratic programming, i.e.,

$$f(x) = \sum_i f_i(x) \qquad f_i(x) = (a_i x - b_i)^2$$

State of the art

synchronous communications:

ADMM (Bertsekas 1997, Boyd 2010, He 2011, Deng 2011, Johansson 2008, Mota 2012,. . .)

asynchronous communications with perfect channel feedback:

ADMM with symmetric gossip (Wei Ozdaglar 2012, Jakovetic 2011, . . .)

asynchronous communications without perfect channel feedback:

Distributed quadratic programming, i.e.,

$$f(x) = \sum_i f_i(x) \qquad f_i(x) = (a_i x - b_i)^2$$

asynchronous communications without perfect channel feedback:

in general ???

In this work

From distributed quadratic programming:

$$f(x) = \sum_i (a_i x - b_i)^2 \quad \Rightarrow \quad x^* = \frac{\frac{1}{N}\sum_i a_i b_i}{\frac{1}{N}\sum_i a_i^2}$$

to $\mathcal{C}^2$ functions with bounded second derivative:

$$f_i(x) \in \mathcal{C}^2 \qquad f_i''(x) > c \quad \Rightarrow \quad x^* = \arg\min_{\widetilde{x}} \sum_i f_i(\widetilde{x})$$

In this work

From distributed quadratic programming:

$$f(x) = \sum_i (a_i x - b_i)^2 \quad \Rightarrow \quad x^* = \frac{\frac{1}{N}\sum_i a_i b_i}{\frac{1}{N}\sum_i a_i^2}$$

to $\mathcal{C}^2$ functions with bounded second derivative:

$$f_i(x) \in \mathcal{C}^2 \qquad f_i''(x) > c \quad \Rightarrow \quad x^* = \arg\min_{\widetilde{x}} \sum_i f_i(\widetilde{x})$$

building block: still average consensus

how to compute an average?

synchronous communications (1):

synchronous consensus: $\boldsymbol{x}(k+1) = P\boldsymbol{x}(k)$ (with P doubly stochastic) (Markov chains ('60s), Seneta 2006, ...)

synchronous communications (1):

synchronous consensus: $\boldsymbol{x}(k+1) = P\boldsymbol{x}(k)$ (with P doubly stochastic) (Markov chains ('60s), Seneta 2006, ...)

synchronous communications (2):

synchronous gossip consensus (Fagnani Zampieri 2008, Boyd et al. 2006, ...)

synchronous communications (1):

synchronous consensus: $\boldsymbol{x}(k+1) = P\boldsymbol{x}(k)$ (with P doubly stochastic) (Markov chains ('60s), Seneta 2006, . . .)

synchronous communications (2):

synchronous gossip consensus (Fagnani Zampieri 2008, Boyd et al. 2006, . . .)

asynchronous communications with perfect channel feedback:

ratio consensus (Bénézit et al. 2010)

synchronous communications (1):

synchronous consensus: $\boldsymbol{x}(k+1) = P\boldsymbol{x}(k)$ (with P doubly stochastic) (Markov chains ('60s), Seneta 2006, ...)

synchronous communications (2):

synchronous gossip consensus (Fagnani Zampieri 2008, Boyd et al. 2006, ...)

asynchronous communications with perfect channel feedback:

ratio consensus (Bénézit et al. 2010)

asynchronous communications without perfect channel feedback:

robust ratio consensus (Dominguez-Garcia et al. 2011)

Ratio consensus

asynchronous communications with perfect channel feedback (Bénézit et al. 2010)

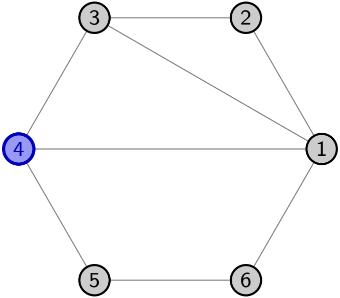
$$
\begin{cases}
\boldsymbol{x}(k+1) = P(k)\boldsymbol{x}(k) \\
x_i(0) = \theta_i \\
\boldsymbol{y}(k+1) = P(k)\boldsymbol{y}(k) \\
y_i(0) = 1
\end{cases}
$$

Ratio consensus

asynchronous communications with perfect channel feedback (Bénézit et al. 2010)

$$\begin{cases} \boldsymbol{x}(k+1) = P(k)\boldsymbol{x}(k) \\ x_i(0) = \theta_i \\ \\ \boldsymbol{y}(k+1) = P(k)\boldsymbol{y}(k) \\ y_i(0) = 1 \end{cases}$$

$$P(k) = \begin{bmatrix} 1 & 0 & 0 & 1/4 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

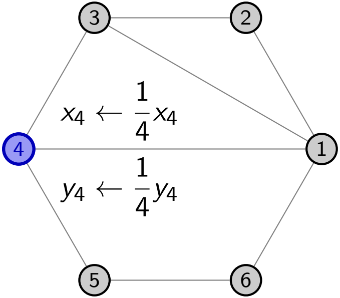


Ratio consensus

asynchronous communications with perfect channel feedback (Bénézit et al. 2010)

$$\begin{cases} \boldsymbol{x}(k+1) = P(k)\boldsymbol{x}(k) \\ x_i(0) = \theta_i \\ \\ \boldsymbol{y}(k+1) = P(k)\boldsymbol{y}(k) \\ y_i(0) = 1 \end{cases}$$

$$P(k) = \begin{bmatrix} 1 & 0 & 0 & 1/4 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

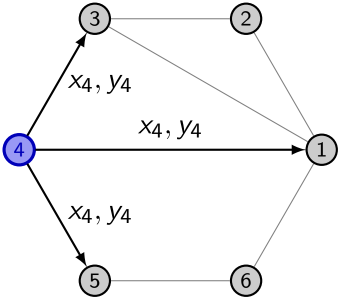


Ratio consensus

asynchronous communications with perfect channel feedback (Bénézit et al. 2010)

$$\begin{cases} \mathbf{x}(k+1) = P(k)\mathbf{x}(k) \\ x_i(0) = \theta_i \\ \\ \mathbf{y}(k+1) = P(k)\mathbf{y}(k) \\ y_i(0) = 1 \end{cases}$$

$$P(k) = \begin{bmatrix} 1 & 0 & 0 & 1/4 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

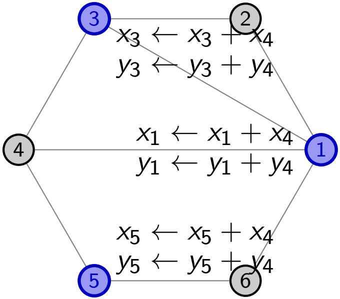


Ratio consensus

asynchronous communications with perfect channel feedback (Bénézit et al. 2010)

$$\begin{cases} \boldsymbol{x}(k+1) = P(k)\boldsymbol{x}(k) \\ x_i(0) = \theta_i \\ \boldsymbol{y}(k+1) = P(k)\boldsymbol{y}(k) \\ y_i(0) = 1 \end{cases}$$

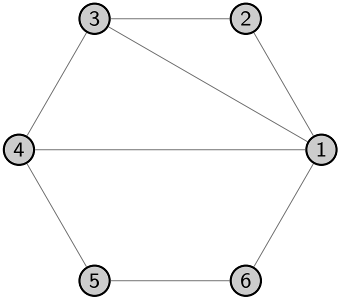
$$P(k) = \begin{bmatrix} 1 & 0 & 0 & 1/4 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$



Ratio consensus

asynchronous communications with perfect channel feedback (Bénézit et al. 2010)

$$\begin{cases} \boldsymbol{x}(k+1) = P(k)\boldsymbol{x}(k) \\ x_i(0) = \theta_i \\ \boldsymbol{y}(k+1) = P(k)\boldsymbol{y}(k) \\ y_i(0) = 1 \end{cases}$$



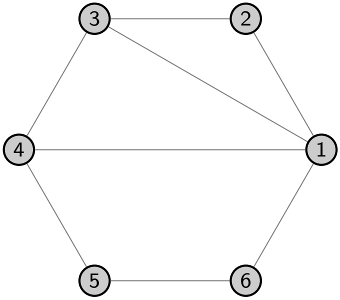
$$P(k) = \begin{bmatrix} 1 & 0 & 0 & 1/4 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{cases} x_i(k) \to \beta_i(k) \sum_j x_i(0) \\ y_i(k) \to \beta_i(k) \sum_j y_i(0) \end{cases}$$

Ratio consensus

asynchronous communications with perfect channel feedback (Bénézit et al. 2010)

$$\begin{cases} \boldsymbol{x}(k+1) = P(k)\boldsymbol{x}(k) \\ x_i(0) = \theta_i \\ \\ \boldsymbol{y}(k+1) = P(k)\boldsymbol{y}(k) \\ y_i(0) = 1 \end{cases}$$



$$P(k) = \begin{bmatrix} 1 & 0 & 0 & 1/4 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{cases} x_i(k) \to \beta_i(k) \sum_j x_i(0) \\ \\ y_i(k) \to \beta_i(k) \sum_j y_i(0) \end{cases} \implies z_i(k) := \frac{x_i(k)}{y_i(k)} \to \frac{\sum_i x_i(0)}{\sum_i y_i(0)} = \frac{1}{N} \sum_i \theta_i$$

Robust ratio consensus

asynch. comm. ***without*** perfect channel feedback (Dominguez-Garcia et al. 2011)

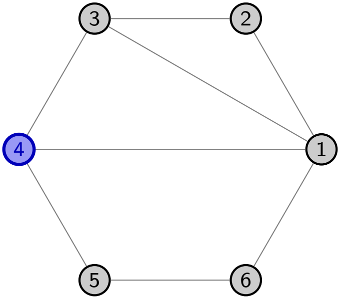
$$
\begin{cases}
\boldsymbol{x}(k+1) = P(k)\boldsymbol{x}(k) \\
x_i(0) = \theta_i \\[1ex]
\boldsymbol{y}(k+1) = P(k)\boldsymbol{y}(k) \\
y_i(0) = 1
\end{cases}
$$

Robust ratio consensus

asynch. comm. ***without*** perfect channel feedback (Dominguez-Garcia et al. 2011)

$$\begin{cases} \boldsymbol{x}(k+1) = P(k)\boldsymbol{x}(k) \\ x_i(0) = \theta_i \\ \boldsymbol{y}(k+1) = P(k)\boldsymbol{y}(k) \\ y_i(0) = 1 \end{cases}$$

$$P(k) = \begin{bmatrix} 1 & 0 & 0 & \mathbf{0} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

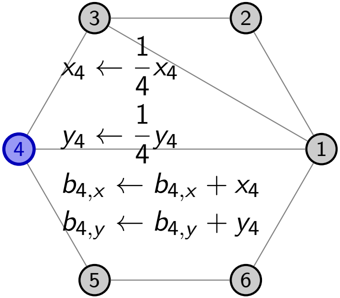


Robust ratio consensus

asynch. comm. ***without*** perfect channel feedback (Dominguez-Garcia et al. 2011)

$$\begin{cases} \boldsymbol{x}(k+1) = P(k)\boldsymbol{x}(k) \\ x_i(0) = \theta_i \\ \\ \boldsymbol{y}(k+1) = P(k)\boldsymbol{y}(k) \\ y_i(0) = 1 \end{cases}$$

$$P(k) = \begin{bmatrix} 1 & 0 & 0 & \mathbf{0} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$



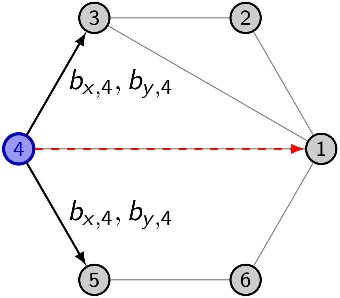
- $b_{i,x}$: total cumulative mass of x_i
- $\beta_{i,x}^{(j)}$: j's local estimate of $b_{i,x}$

Robust ratio consensus

asynch. comm. ***without*** perfect channel feedback (Dominguez-Garcia et al. 2011)

$$\begin{cases} \boldsymbol{x}(k+1) = P(k)\boldsymbol{x}(k) \\ x_i(0) = \theta_i \\ \boldsymbol{y}(k+1) = P(k)\boldsymbol{y}(k) \\ y_i(0) = 1 \end{cases}$$

$$P(k) = \begin{bmatrix} 1 & 0 & 0 & \mathbf{0} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$



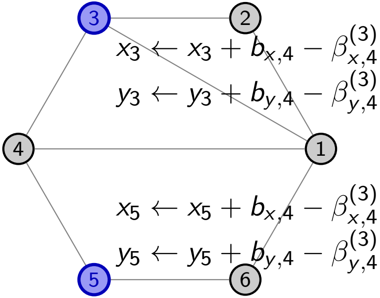
- $b_{i,x}$: total cumulative mass of x_i
- $\beta_{i,x}^{(j)}$: j's local estimate of $b_{i,x}$

Robust ratio consensus

asynch. comm. ***without*** perfect channel feedback (Dominguez-Garcia et al. 2011)

$$\begin{cases} \boldsymbol{x}(k+1) = P(k)\boldsymbol{x}(k) \\ x_i(0) = \theta_i \\ \boldsymbol{y}(k+1) = P(k)\boldsymbol{y}(k) \\ y_i(0) = 1 \end{cases}$$

$$P(k) = \begin{bmatrix} 1 & 0 & 0 & \mathbf{0} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$



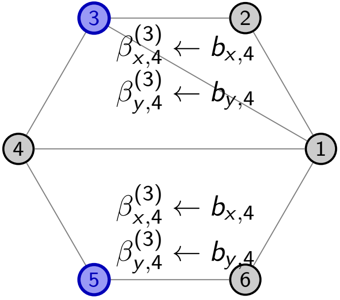
- $b_{i,x}$: total cumulative mass of x_i
- $\beta^{(j)}_{i,x}$: j's local estimate of $b_{i,x}$

Robust ratio consensus

asynch. comm. ***without*** perfect channel feedback (Dominguez-Garcia et al. 2011)

$$\begin{cases} \boldsymbol{x}(k+1) = P(k)\boldsymbol{x}(k) \\ x_i(0) = \theta_i \\ \boldsymbol{y}(k+1) = P(k)\boldsymbol{y}(k) \\ y_i(0) = 1 \end{cases}$$

$$P(k) = \begin{bmatrix} 1 & 0 & 0 & \mathbf{0} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$



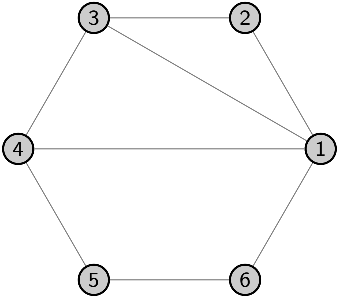
- $b_{i,x}$: total cumulative mass of x_i
- $\beta^{(j)}_{i,x}$: j's local estimate of $b_{i,x}$

Robust ratio consensus

asynch. comm. ***without*** perfect channel feedback (Dominguez-Garcia et al. 2011)

$$\begin{cases} \boldsymbol{x}(k+1) = P(k)\boldsymbol{x}(k) \\ x_i(0) = \theta_i \\ \boldsymbol{y}(k+1) = P(k)\boldsymbol{y}(k) \\ y_i(0) = 1 \end{cases}$$

$$P(k) = \begin{bmatrix} 1 & 0 & 0 & \mathbf{0} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$



$$z_i(k) = \frac{x_i(k)}{y_i(k)} \to \frac{1}{N}\sum_j \theta_i$$

- $b_{i,x}$: total cumulative mass of x_i
- $\beta_{i,x}^{(j)}$: j's local estimate of $b_{i,x}$

distributed Newton-Raphson

Robust distributed asynchronous quadratic programming

Robust distributed asynchronous quadratic programming

Initialization

$$f_i(x) = \frac{1}{2}\left(a_i x - b_i\right)^2 \quad \Rightarrow \quad \begin{cases} y_i & \leftarrow a_i b_i \\ z_i & \leftarrow a_i^2 \end{cases}$$

Robust distributed asynchronous quadratic programming

Initialization

$$f_i(x) = \frac{1}{2}\left(a_i x - b_i\right)^2 \quad \Rightarrow \quad \begin{cases} y_i & \leftarrow a_i b_i \\ z_i & \leftarrow a_i^2 \end{cases}$$

transmitting node

$$y_i \leftarrow \frac{1}{|\mathcal{N}_i^{\text{out}}| + 1} y_i$$

$$z_i \leftarrow \frac{1}{|\mathcal{N}_i^{\text{out}}| + 1} z_i$$

$$x_i \leftarrow (1 - \varepsilon) x_i + \varepsilon \frac{y_i}{z_i}$$

$$b_{i,y} \leftarrow b_{i,y} + y_i$$

$$b_{i,z} \leftarrow b_{i,z} + z_i$$

receiving node

$$y_j \leftarrow y_j + b_{i,y} - \beta_{i,y}^{(j)}$$

$$z_j \leftarrow z_j + b_{i,z} - \beta_{i,z}^{(j)}$$

$$x_j \leftarrow (1 - \varepsilon) x_j + \varepsilon \frac{y_j}{z_j}$$

$$\beta_{i,y}^{(j)} \leftarrow b_{i,y}$$

$$\beta_{i,z}^{(j)} \leftarrow b_{i,z}$$

Assumptions

- fixed, strongly connected and directed network
- exponential i.i.d. waiting times between local broadcasts
- $\mathbb{P}\left[\text{unsuccessful communications}\right] < 1$
- $\varepsilon \in (0, 1]$

Proposition

$$\mathbb{P}\left[\lim_{t\to\infty} x_i(t) = x^*\right] = 1 \qquad \forall i$$

robust distributed asynchronous quadratic programming

↓

robust distributed asynchronous Newton-Raphson *robust distributed asynchronous quadratic programming*

↓

robust distributed asynchronous Newton-Raphson

TODOs:

1. modify the initialization
2. modify the transmission / reception
3. prove the convergence properties

Robust distributed asynchronous Newton-Raphson

Initialization for quadratic programming

$$f_i(x) = \frac{1}{2}\left(a_i x - b_i\right)^2 \quad \Rightarrow \quad \begin{cases} y_i & \leftarrow a_i b_i \\ z_i & \leftarrow a_i^2 \end{cases}$$

Initialization for Newton-Raphson

$$f_i(x) \in \mathcal{C}^2, \quad f_i''(x) > c$$

$$\Rightarrow \quad \begin{cases} x_i & \leftarrow x^o \\ y_i = g_i^{\text{old}} = g_i & \leftarrow f_i''\left(x^o\right) x^o - f_i'\left(x^o\right) \\ z_i = h_i^{\text{old}} = h_i & \leftarrow f_i''\left(x^o\right) \end{cases}$$

Robust distributed asynchronous Newton-Raphson

Transmission for quadratic programming

$$
\begin{aligned}
y_i &\leftarrow \frac{1}{|\mathcal{N}_i^{\text{out}}|+1} y_i \\
z_i &\leftarrow \frac{1}{|\mathcal{N}_i^{\text{out}}|+1} z_i \\
x_i &\leftarrow (1-\varepsilon)x_i + \varepsilon \frac{y_i}{z_i}
\end{aligned}
$$

$$
\begin{aligned}
b_{i,y} &\leftarrow b_{i,y} + y_i \\
b_{i,z} &\leftarrow b_{i,z} + z_i
\end{aligned}
$$

Transmission for Newton-Raphson

$$
\begin{aligned}
y_i &\leftarrow \frac{1}{|\mathcal{N}_i^{\text{out}}|+1} \left[y_i + g_i - g_i^{\text{old}} \right] \\
z_i &\leftarrow \frac{1}{|\mathcal{N}_i^{\text{out}}|+1} \left[z_i + h_i - h_i^{\text{old}} \right] \\
x_i &\leftarrow (1-\varepsilon)x_i + \varepsilon \frac{y_i}{[z_i]_c}
\end{aligned}
$$

$$
\begin{aligned}
b_{i,y} &\leftarrow b_{i,y} + y_i \\
b_{i,z} &\leftarrow b_{i,z} + z_i \\
g_i^{\text{old}} &\leftarrow g_i \\
h_i^{\text{old}} &\leftarrow h_i \\
g_i &\leftarrow f_i''(x_i)x_i - f_i'(x_i) \\
h_i &\leftarrow f_i''(x_i)
\end{aligned}
$$

Robust distributed asynchronous Newton-Raphson

Reception for quadratic programming

$$
\begin{aligned}
y_j &\leftarrow y_j + b_{i,y} - \beta_{i,y}^{(j)} \\
z_j &\leftarrow z_j + b_{i,z} - \beta_{i,z}^{(j)} \\
x_j &\leftarrow (1-\varepsilon)x_j + \varepsilon \frac{y_j}{z_j}
\end{aligned}
\qquad
\begin{aligned}
\beta_{i,y}^{(j)} &\leftarrow b_{i,y} \\
\beta_{i,z}^{(j)} &\leftarrow b_{i,z}
\end{aligned}
$$

Reception for Newton-Raphson

$$
\begin{aligned}
y_j &\leftarrow y_j + b_{i,y} - \beta_{i,y}^{(j)} + g_j - g_j^{\text{old}} \\
z_j &\leftarrow z_j + b_{i,z} - \beta_{i,z}^{(j)} + h_j - h_j^{\text{old}} \\
x_j &\leftarrow (1-\varepsilon)x_j + \varepsilon \frac{y_j}{[z_j]_c}
\end{aligned}
\qquad
\begin{aligned}
\beta_{i,y}^{(j)} &\leftarrow b_{i,y} \\
\beta_{i,z}^{(j)} &\leftarrow b_{i,z} \\
g_j^{\text{old}} &\leftarrow g_j \\
h_j^{\text{old}} &\leftarrow h_j \\
g_j &\leftarrow f_i''(x_j)x_i - f_i'(x_j) \\
h_j &\leftarrow f_i''(x_j)
\end{aligned}
$$

Convergence properties

Assumptions

- $f_i \in \mathcal{C}^2, \quad f_i''(x) > c$
- fixed, strongly connected and directed network
- communications are persistent

  (i.e., at least 1 communication in every $[t, t+\tau]$*)*
- bounded packet losses

  (i.e., number of consecutive failures is limited)

Proposition

$\exists\ B_\delta(x^*)$ and $\varepsilon_c \in \mathbb{R}_+$ s.t. if $x^o \in B_\delta$ and $0 < \varepsilon < \varepsilon_c$ then

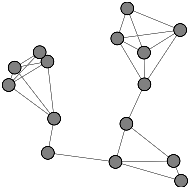
$$|x_i(k) - x^*| \leq c\lambda^k \qquad \forall i$$

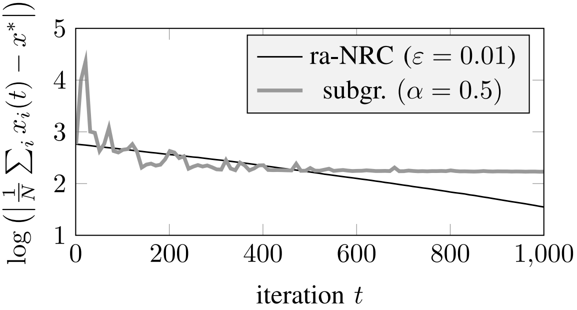
for opportune $c \in \mathbb{R}_+$ and $\lambda < 1$

Numerical experiments

algorithms tuned with their best parameters and packet loss probability $p = 0.1$

$$f_i(x) = \frac{(y_i - \langle \boldsymbol{\chi}_i, \widetilde{\boldsymbol{x}} \rangle)^2}{|y_i - \langle \boldsymbol{\chi}_i, \widetilde{\boldsymbol{x}} \rangle| + \beta} + \gamma \, \|\boldsymbol{x}\|_2^2$$





Future directions

- tuning ε online
- partition-based updates of $\boldsymbol{x}$
- equality constraints $A\boldsymbol{x} = \boldsymbol{b}$

Analysis of Newton-Raphson Consensus for multi-agent convex optimization under asynchronous and lossy communications

Ruggero Carli Giuseppe Notarstefano
Luca Schenato **Damiano Varagnolo**

CDC 2015 - Osaka

December 14, 2015

damiano.varagnolo@ltu.se

entirely written in LaTeX2ε using Beamer and Tik Z