Control of HVAC Systems via Scenario-based Explicit MPC

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Abstract— Improving energy efficiency of Heating, Ventilation and Air Conditioning (HVAC) systems is a primary objective for the society. Model Predictive Control (MPC) techniques for HVAC systems have recently received particular attention, since they can naturally account for several factors, such as weather and occupancy forecasts, comfort ranges and actuation constraints. Developing effective MPC based control strategies for HVAC systems is nontrivial, since buildings dynamics are nonlinear and affected by various uncertainties. Further, the complexity of the MPC problem and the burden of on-line computations can lead to difficulties in integrating this scheme into a building management system.

We propose to address this computational issue by designing a scenario-based explicit MPC strategy, i.e., a controller that is simultaneously based on explicit representations of the MPC feedback law and accounts for uncertainties in the occupancy patterns and weather conditions by using the scenarios paradigm. The main advantages of this approach are the absence of a-priori assumptions on the distributions of the uncertain variables, the applicability to any type of building, and the limited on-line computational burden, enabling practical implementations on low-cost hardware platforms.

We illustrate the practical implementation of the proposed explicit MPC controller on a room of a university building, showing its effectiveness and computational tractability.

I. INTRODUCTION

Heating, Ventilation and Air Conditioning (HVAC) systems play a fundamental role in maintaining acceptable indoor comfort levels; reports indicate that HVAC systems in developed countries contribute for approximately one fifth of the total national energy usages [1]. Current practice shows its limits, with potential energy savings achievable by using systematic building management being estimated from 5% to 30% of the total consumptions [2], [3]. An effective controller for HVAC systems should incorporate time-dependent energy costs, bounds on the control actions, comfort requirements, as well as account for system uncertainties, e.g., weather conditions and occupancy. A natural scheme that achieves the systematic integration of all the aforementioned elements is the Model Predictive Control (MPC) [4].

Simulations in [5], [6], as well as the experimental results on real buildings reported in [7], [8], [9], show that MPC schemes can yield better comfort levels and energy use performance than current practices.

In particular, successful implementations are likely to be based on stochastic MPC schemes with probabilistic constraints, i.e., the so called *chance constrained* MPCs [10]. Indeed, current standards explicitly account for the possibility of comfort violations, stating that the probability of these violations should not exceed certain levels [11].

Literature review: there is a vast literature on stochastic MPC schemes for HVAC control, e.g., [12], [13], [14]. Nonetheless, to simplify the tractability of the MPC problem the aforementioned works restrict the disturbances to have Gaussian distributions. Furthermore, these approaches do not explicitly consider the uncertainty in the occupancy when controlling the CO_2 levels.

These two issues are addressed in [15], [16], where authors consider scenario-based approximations of a chance constrained MPC problem that accounts for uncertainty by extracting the scenarios from general probability distributions, thus not restricted to be Gaussian [17].

The numerical simulations performed in these works suggest that scenarios-based techniques outperform other predictive methods.

However, all these MPC strategies come with computational demand issues, since they need to run on-line optimization algorithms to solve the MPC optimization problem. In addition, scenario-based approaches requires to generate a large number of scenarios on-line. The drawback associated to this on-line computational burden is that implementation on cheap hardware platforms (such as in Programmable Logic Controllers (PLCs)) and integration in Building Automation Systems (BASs) may become prohibitive. A possible solution is to determine explicitly, off-line, the solution of the MPC optimization problem as a function of the current building state. This leads to explicit solutions of the MPC problem, which are obtained by solving multiparametric programs [18], [19], where the state vector is treated as a vector of parameters. The optimal control profile is available as a Piecewise Affine (PWA) function of the initial state.

By doing so, the computation associated with MPC reduces to a function evaluation problem, which requires simple software and cheap hardware. Besides these practical benefits, the explicit control structure also may provide insights on the structure of the control action. However, the applicability of these methods is limited to fairly small problems because of memory requirements and off-line computational times.

The few works that provide explicit MPC control laws for building temperature control, [20], [21], show, through simulations, that the explicit MPC can outperform control logics used in the buildings industry and provide performance

comparable to a controller assuming perfect knowledge of future disturbances.

Statement of Contributions: with respect to the state-of-the-art, we: *i)* compute the explicit state-feedback solution of a Scenario-based Model Predictive Control (SMPC) for both temperature and CO_2 levels control;

Outline of the manuscript: Section II present the model of the building, while Section III describes the novel controller. Section IV provides and discusses experimental results, while Section V eventually summarizes our conclusions and proposes some future directions.

II. BUILDING MODELING

The following thermal model is tailored for the explicit MPC paradigm, and trades-off the accuracy of the predictions with the numerical tractability of the control problem. The model is required to capture the main room temperature dynamics while keeping the number of states limited. The requirement on the size of the state is mandatory, since the off-line computational time for the computation of the MPC solution increases dramatically with the problem dimension.

We adopt a two states thermal model of a single thermal zone (or room) using the quantities defined in Table I. Extensions to whole building enclosures or to room networks are conceptually straightforward and motivated by the fact that the overall building energy use is commonly computed as the sum the energy usages of the single thermal zones [22].

A_{win}	windows area
C	heat emission (per occupant)
$C_{CO_2,i}$	CO_2 concentration of the air inlet
$C_{p,a}$	air specific heat capacity
G	solar heat gain coefficient
g_{CO_2}	CO_2 emitted per occupant
I	incident solar radiation on the windows
$\dot{m}_{\text{vent}}$	ventilation massflow
N_p	number of room occupants
R_{ae}	window thermal resistance between the indoor and the outdoor temperature
R_{aw}	thermal resistance between the indoor air and the walls
R_{we}	thermal resistance between the wall and outdoor air
T_{air}	average temperature of the indoor air
T_{aver}	equivalent average temperature (accounts for heat transfer though the walls and external air temperatures)
T_{ext}	outdoor temperature
T_{mr}	mean radiant temperature of the radiators
T_{sa}	air inlet temperature
T_{wall}	average temperature of the walls
$\Delta T_{\text{h,rad}}$	$T_{\text{mr}} - T_{\text{air}}$
ΔT_{vent}	$T_{\text{sa}} - T_{\text{air}}$

TABLE I

QUANTITIES INVOLVED IN THE BUILDING MODELS.

A. Thermal and CO$_2$ models

We model a thermal zone as a network of two nodes, one accounting for the dynamics of the air within the zone, and the other one accounting for the dynamics of the walls. In formulas,

$$\begin{cases} C_a \dot{T}_{\text{air}} = \dfrac{T_{\text{wall}} - T_{\text{air}}}{R_{\text{aw}}} + \dfrac{T_{\text{ext}} - T_{\text{air}}}{R_{\text{ae}}} \\ \qquad\qquad + CN_p + GA_{\text{win}}I + Q_{\text{vent}} + Q_{\text{heat}} \quad \text{(1a)} \\ C_w \dot{T}_{\text{wall}} = \dfrac{T_{\text{air}} - T_{\text{wall}}}{R_{\text{aw}}} + \dfrac{T_{\text{aver}} - T_{\text{wall}}}{R_{\text{we}}} \quad \text{(1b)} \end{cases}$$

where

- $Q_{\text{vent}} = \dot{m}_{\text{vent}} C_{\text{pa}} \Delta T_{\text{vent}}$ represents the heat flow due to the ventilation system;
- $Q_{\text{heat}} = A_{\text{rad}} h_{\text{rad}} \Delta T_{\text{h,rad}}$ is the heating flow due to the heating system (e.g., radiators);
- T_{aver} is an equivalent temperature accounting for the different temperatures of the zone walls calculated as

$$T_{\text{aver}} = \frac{\sum_{i=1}^{6} T_{w,\text{out}}^i A^i}{\sum_{i=1}^{6} A^i} \qquad (2)$$

i.e., as the average of the temperatures on the external side of the i-th wall, $T_{w,\text{out}}$, weighted by the i-th wall area, A^i;

As for the dynamics of the CO_2 concentration in the room, we consider the model in [23], i.e.,

$$\dot{C}_{CO_2} = \left(\dot{m}_{\text{vent}}^{CO_2} C_{CO_2,i} + g_{CO_2} N_p \right) - \dot{m}_{\text{vent}} C_{CO_2} \qquad (3)$$

where the inlet air concentration $C_{CO_2,i}$ is assumed equal to outdoor CO_2 concentration levels.

B. Control-Oriented Modeling

Nonlinearities in the dynamics (1a) and (3) can lead to intractable multiparametric optimization problems. To address this issue, we derive linear reformulations of (1a) and (3), which are then discretized with opportune sampling time ΔT. In the next paragraphs we will describe the obtained state-space linear models of CO_2 concentration and room temperature levels.

State-space model for the CO$_2$ dynamics: we consider the discrete-time Linear Time Invariant (LTI) system

$$\begin{aligned} x_{CO_2}(k+1) &= a x_{CO_2}(k) + b u_{CO_2}(k) + e w_{CO_2}(k) \\ y_{CO_2}(k) &= x_{CO_2}(k) \end{aligned} \qquad (4)$$

where:

- x_{CO_2}, i.e., the state, is equal to $C_{CO_2} - C_{CO_2,i}$;
- w_{CO_2}, i.e., the disturbance, is equal to the number of occupants N_p;
- u_{CO_2}, i.e., the control input, is defined as $\dot{m}_{\text{vent}}^{CO_2} \cdot x_{CO_2}$, hence it hides the bilinear term in (3). Once $u_{CO_2}(k)$ is computed and $x_{CO_2}(k)$ is known, one derives immediately $\dot{m}_{\text{vent}}^{CO_2}(k)$ by a simple inversion. Notice that, to meet the physical bounds on the original control input $\dot{m}_{\text{vent}}^{CO_2}(k)$, $u_{CO_2}(k)$ must satisfy

$$\dot{m}_{\text{vent}}^{\min} \cdot x_{CO_2}(k) \le u_{CO_2}(k) \le \dot{m}_{\text{vent}}^{\max} \cdot x_{CO_2}(k). \quad (5)$$

State-space model for the thermal dynamics: we consider the discrete-time linear system

$$\begin{aligned} x_{\text{T}}(k+1) &= A_{\text{T}} x_{\text{T}}(k) + B_{\text{T}}(k) u_{\text{T}}(k) + E_{\text{T}} w_{\text{T}}(k) \\ y_{\text{T}}(k) &= C_{\text{T}} x_{\text{T}}(k) \end{aligned} \qquad (6)$$

where

- $x_{\text{T}}(k)$, i.e., the state, is a vector containing the temperatures of the indoor air and the walls;
- $w_{\text{T}}(k)$, i.e., the disturbance, is a vector containing the outdoor temperature, the solar radiation and internal

heat gains are due to occupancy, equipments and light-ings;

- $y_T(k)$, i.e., the output, is the measured indoor temperature at time k;
- $u_T(k)$, i.e., the control input, is defined as $\left[\Delta T_h(k), \Delta T_c(k), \Delta u_h(k), \Delta u_c(k), \Delta T_{h,\text{rad}}(k)\right]$.

The input matrix $B_T(k)$ is time varying since it depends on $\dot{m}_{\text{vent}}^{\text{CO}_2}(k)$, as illustrated in the following.

In the thermal model (1), the control inputs are the temperature of the supplied air, T_{sa}, the mean radiant temperature of the radiators, T_{mr}, and the massflow $\dot{m}_{\text{vent}}$. These inputs allow to control two different heat flows, $Q_{\text{vent}} = \dot{m}_{\text{vent}} C_{\text{pa}}(T_{\text{sa}} - T_{\text{air}})$ and $Q_{\text{heat}} = A_{\text{rad}} h_{\text{rad}} \Delta T_{\text{h,rad}}$. We assume the convention that Q_{vent} is positive when the system requires heating, and negative when it requires cooling.

Notice then the presence of the bilinear term $\dot{m}_{\text{vent}} \Delta T_{\text{vent}}$ in Q_{vent}. Notice moreover that $\dot{m}_{\text{vent}}$ must be at least equal to $\dot{m}_{\text{vent}}^{\text{CO}_2}$, the latter representing the minimum massflow needed to maintain optimal CO_2 levels.

To hide the bilinear term above and simultaneously account for the minimum requirements on the massflow levels we introduce the nonnegative variables ΔT_h, ΔT_c, Δu_h and Δu_c such that

$$\Delta T_h - \Delta T_c = T_{\text{sa}} - T_{\text{air}}$$
$$\Delta u_h - \Delta u_c = \Delta \dot{m}_{\text{vent}} \Delta T_{\text{vent}},$$

where $\Delta \dot{m}_{\text{vent}} := \dot{m}_{\text{vent}} - \dot{m}_{\text{vent}}^{\text{CO}_2}$ represents the additional massflow required for guaranteeing the thermal comfort. Summarizing, the controllable inputs for (6) are

$$u_T(k) = \left[\Delta T_h(k) \ \ \Delta T_c(k) \ \ \Delta u_h(k) \ \ \Delta u_c(k) \ \ \Delta T_{h,\text{rad}}(k)\right].$$

With these new variables Q_{vent} can be written as

$$Q_{\text{vent}} = \dot{m}_{\text{vent}}^{\text{CO}_2} C_{\text{pa}}(\Delta T_h - \Delta T_c) + C_{\text{pa}}(\Delta u_h - \Delta u_c).$$

Consider then that the control objective aims to minimize the thermal energy use, that may be required for either cooling or heating the space. Hence, instead of considering Q_{vent}, that can be either positive or negative, we need to model in the objective function of the MPC optimization problem its absolute value $|Q_{\text{vent}}|$, that can then be written as

$$|Q_{\text{vent}}| = \dot{m}_{\text{vent}}^{\text{CO}_2} C_{\text{pa}}(\Delta T_h + \Delta T_c) + C_{\text{pa}}(\Delta u_h + \Delta u_c).$$

Once $\dot{m}_{\text{vent}}^{\text{CO}_2}(k)$ and $u_T(k)$ have been computed, $T_{\text{sa}}(k)$, $T_{\text{mr}}(k)$ and $\dot{m}_{\text{vent}}(k)$ can be easily computed by simple inversions. Consider also that the input-to-state matrix $B_T(k)$ is time varying, since it accounts for the time-varying $\dot{m}_{\text{vent}}^{\text{CO}_2}(k)$.

Eventually we can account for the physical bounds on the original control inputs at each time step k by accounting for the bounds

$$\begin{cases} T_{\text{sa}}^{\min} - T_{\text{air}}(k) \leq \Delta T_h(k) - \Delta T_c(k) \leq T_{\text{sa}}^{\max} - T_{\text{air}}(k) & (7a) \\ \Delta u_h(k) \leq \Delta \dot{m}_{\text{vent}}^{\max}(k) \Delta T_h(k) & (7b) \\ \Delta u_c(k) \leq \Delta \dot{m}_{\text{vent}}^{\max}(k) \Delta T_c(k) & (7c) \end{cases}$$

where $\Delta \dot{m}_{\text{vent}}^{\max}(k) := \dot{m}_{\text{vent}}^{\max} - \dot{m}_{\text{vent}}^{\text{CO}_2}(k)$.

III. SCENARIO-BASED EXPLICIT MPC FOR HVAC SYSTEMS

We detail the proposed control scheme with Section III-A, which describes the overall architecture and outlines the algorithm for the on-line implementation of the explicit state-feedback control law, and with Section III-B, which then provides the control objectives and formulate the corresponding MPC problem.

A. Control Architecture and On-line Implementation

Figure 1 depicts the architecture of the implemented control system. The indoor temperature and air CO_2 levels are measured and then controlled through the ventilation system and radiators. These devices are actuated using low-level Proportional Integrative (PI) controllers, which setpoints are computed by means of our explicit Explicit Scenario-based Model Predictive Control (ESMPC) scheme.
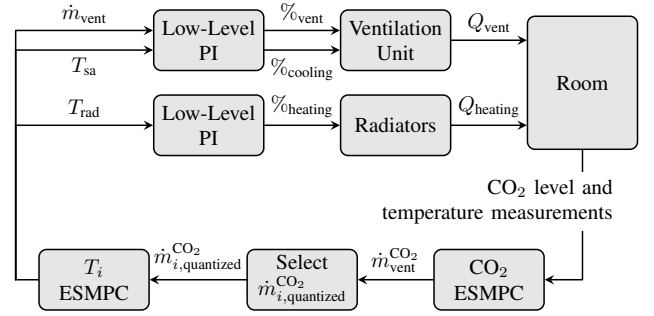

Fig. 1. Architecture of the control system implemented on the testbed.

To improve the computational tractability of the overall control problem, we take advantage of the independence of the CO_2 dynamics from the thermal ones and the priority of CO_2 comfort. This allows us to consider two separated multiparametric subproblems, discussed in Section III-B: *i)* the CO_2-ESMPC problem (minimize the energy used to keep CO_2 levels in given comfort bounds); *ii)* the T-ESMPC problem (minimize energy use to keep temperature levels in given comfort bounds while preserving the minimum massflow computed by the CO_2-ESMPC controller).

By solving the CO_2-ESMPC and the T-ESMPC problems we obtain explicit state-feedback laws that are expressed as PWA function of the state x [18], [19]. Hence, the CO_2-ESMPC and T-ESMPC control laws have respectively the form

$$\begin{aligned} u^{E\text{CO}_2}(x) &= Q_i^{E\text{CO}_2} x + q_i^{E\text{CO}_2} & \text{if } H_i^{E\text{CO}_2} x \leq K_i^{E\text{CO}_2} \\ u^{ET}(x) &= Q_j^{ET} x + q_j^{ET} & \text{if } H_j^{ET} x \leq K_j^{ET} \end{aligned}$$

where the polyhedral sets $\mathcal{X}_i^{E\text{CO}_2} := \left\{ H_i^{E\text{CO}_2} x \leq K_i^{E\text{CO}_2} \right\}$ and $\mathcal{X}_j^{ET} := \left\{ H_j^{ET} x \leq K_j^{ET} \right\}$, with $j = 1 \ldots N^{ET}$ are partitions of the set of states, with $i = 1, \ldots, N^{E\text{CO}_2}$ and $j = 1, \ldots, N^{ET}$.

To guarantee that the overall procedure satisfies the requirements on the minimum massflow needed to maintain comfortable CO_2 levels, we proceed as follows: the set of

admissible values of the massflow is partitioned using standard algorithms (e.g., Lloyd algorithm [24]) and Q quantized values are computed, $\dot{m}_{1,\text{quantized}}^{\text{CO}_2}, \ldots, \dot{m}_{Q,\text{quantized}}^{\text{CO}_2}$. Then, for each $\dot{m}_{i,\text{quantized}}^{\text{CO}_2}$, a corresponding T_i-ESMPC problem that takes $\dot{m}_{i,\text{quantized}}^{\text{CO}_2}$ as lower bound on the massflow is solved [15].

The scenario-based controller can be then implemented on-line according to Algorithm 1.

Algorithm 1 On-line Implementation

1: **for** $k = 1, 2, \ldots$ **do**
2:     measure $x_{\text{CO}_2}(k)$ and $x_{\text{T}}(k)$
3:     compute $u^{E\text{CO}_2}\big(x_{\text{CO}_2}(k)\big)$ and derive $\dot{m}_{\text{vent}}^{\text{CO}_2}(k)$
4:     identify the quantized value $\dot{m}_{i,\text{quantized}}^{\text{CO}_2}(k)$
5:     compute $u^{ET}\big(x_{\text{T}}(k)\big)$ by using the solution of the T_i-ESMPC problem and derive the setpoints $\big(\dot{m}_{\text{vent}}(k), T_{\text{sa}}(k), T_{\text{mr}}(k)\big)$
6:     send the computed setpoints to the low-level PI controllers
7: **end for**

B. Control Objective

The proposed control strategy aims at optimizing HVAC operations so that the thermal energy use is minimized while indoor temperature and CO_2 levels are kept in given comfort ranges.

As suggested in the section above, we formulate two problems, the CO_2-ESMPC problem and the T-ESMPC problem. Since both problems consider discrete-time linear models and consider hard constraints on the inputs and probabilistic constraints on the outputs (CO_2 levels and temperature respectively), we consider an MPC problem for the control of discrete-time systems of the form

$$x(k+1) = Ax(k) + B(k)u(k) + Ew(k)$$
$$y(k) = Cx(k), \tag{8}$$

where $x(k) \in \mathcal{R}^n$ is the state, $u(k) \in \mathcal{R}^m$ is the control input, $w(k) \in \mathcal{R}^r$ is the stochastic disturbance and $y(k) \in \mathcal{R}^p$ is the output. Indeed (8) represents either (4) or (6), depending on the controller under consideration (CO_2-ESMPC or T-ESMPC).

At each time step k we consider a prediction horizon N and define

$$\boldsymbol{x} := \big[x(k+1|k)^{\text{T}}, \ldots, x(k+N|k)^{\text{T}}\big]^{\text{T}},$$
$$\boldsymbol{u} := \big[u(k)^{\text{T}}, \ldots, u(k+N-1)^{\text{T}}\big]^{\text{T}},$$
$$\boldsymbol{y} := \big[y(k+1|k)^{\text{T}}, \ldots, y(k+N|k)^{\text{T}}\big]^{\text{T}},$$
$$\boldsymbol{w} := \big[w(k)^{\text{T}}, \ldots, w(k+N-1)^{\text{T}}\big]^{\text{T}},$$

where $x(k+i+1|k) = Ax(k+i|k) + Bu(k+i) + Ew(k+1)$ and $y(k+i|k) = Cx(k+i|k)$ are the i-steps-ahead state and output predictions. The dynamics of the predicted output can be written in compact form as a function of the initial state $x(k)$, i.e., as

$$\boldsymbol{y} = \boldsymbol{C}_A x(k) + \boldsymbol{C}_B \boldsymbol{u} + \boldsymbol{C}_E \boldsymbol{w} \tag{9}$$

where the prediction dynamics matrices $\boldsymbol{C}_A, \boldsymbol{C}_B, \boldsymbol{C}_E$ can be easily derived from (8).

We then assume three types of constraints: *i)* inputs constraints of the form $u^{\min} \leq Fu(k) \leq u^{\max}$; constraints (7b) and (7c) can be expressed as input constraints; *ii)* comfort constraints on the outputs of the form $y^{\min} \leq y(k) \leq y^{\max}$ [1]); *iii)* mixed constraints on outputs and disturbances of the form $G_y y(k) + G_u u(k) \leq g$; constraints (5) and (7a) can be expressed as mixed constraints.

Given the fact that both the constraints and dynamics (9) are linear, we can compactly write constraints on the inputs, the outputs and mixed constraints as

$$\boldsymbol{G}_{xk} x(k) + \boldsymbol{G}_u \boldsymbol{u} + \boldsymbol{G}_w \boldsymbol{w} \leq \boldsymbol{g}$$
$$\boldsymbol{F}\boldsymbol{u} \leq \boldsymbol{f}, \tag{10}$$

with opportune $\boldsymbol{F} \in \mathcal{R}^{q \times mN}$, $\boldsymbol{f} \in \mathcal{R}^q$, $\boldsymbol{G}_w \in \mathcal{R}^{r \times pN}$, $\boldsymbol{G}_u \in \mathcal{R}^{r \times mN}$, $\boldsymbol{G}_{xk} \in \mathcal{R}^{r \times n}$ and $\boldsymbol{g} \in \mathcal{R}^r$.

We then assume the possibility of violating the comfort bounds on the indoor temperature and CO_2 levels with a predefined probability, i.e., formulate output constraints as the following *chance-constrains*

$$\mathbb{P}\Big[\boldsymbol{G}_{xk} x(k) + \boldsymbol{G}_u \boldsymbol{u} + \boldsymbol{G}_w \boldsymbol{w} \leq \boldsymbol{g}\Big] \geq 1 - \alpha$$

with $\alpha \in [0, 1]$ being the violation probability level. In these formulations α represents a tradeoff between performance and constraint satisfaction.

As mentioned above, our control strategy aims at minimizing the thermal energy use. This means that the cost function in the proposed MPC problem formulations is the energy use over the whole prediction horizon, e.g., $\sum_{k=0}^{N-1}\big(|Q_{\text{vent}}| + Q_{\text{heat}}\big)$ in the T-ESMPC problem. Denoting by $\boldsymbol{c}^{\text{T}}\boldsymbol{u}\Delta k$, $\boldsymbol{c} \in \mathbb{R}^{mN}$, the cost vector for both the T-ESMPC and the CO_2-ESMPC and Δk the sampling period, the general control problem can be formally stated as

$$\min_{\boldsymbol{u}} \quad \boldsymbol{c}^{\text{T}}\boldsymbol{u}\Delta k$$
$$\text{s.t.} \quad \mathbb{P}\Big[\boldsymbol{G}_{xk} x(k) + \boldsymbol{G}_u \boldsymbol{u} + \boldsymbol{G}_w \boldsymbol{w} \leq \boldsymbol{g}\Big] \geq 1 - \alpha \tag{11}$$
$$\boldsymbol{F}\boldsymbol{u} \leq \boldsymbol{f}.$$

Chance constrained problems like (11) are generally intractable unless the uncertainties follow specific distributions, e.g., Gaussian [10].

Since Gaussian assumptions are rather restrictive, specially to handle strongly heteroskedastic processes like occupancy patterns, we propose to overcome this limitation and obtain a solvable MPC problem by applying *randomized* approaches [25], [26]. These approaches do not require to specify the probability distribution of the uncertainties, but just the possibility to extract i.i.d. samples, also called *scenarios*, from their distribution. In [25], [27] the minimum number of samples guaranteeing that considering constraints (**??**) will lead to a feasible solution for Problem (11) with a given confidence level is provided. Recently the scenario approach has been applied in an MPC framework [28], where authors have shown that bounds on the number of scenarios

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[1]We incidentally notice that comfort bounds are usually time-varying.

can be improved if the problem contains multiple chance constraints (e.g. MPC problems), or if a chance constraint belongs to a special class of constraints (e.g. linear or quadratic constraints). In these cases the conservativeness of the scenario solution can be reduced. Moreover, the authors in [29] show that the theory can be extended to the case where a given number of sampled constraints are discarded a posteriori, so that the objective value achieved by the scenario solution can be improved.

Hence, as in [9], we model the uncertainty in occupancy levels, outdoor temperatures, and incident solar radiation as independent stochastic processes and estimate their probability distributions from historical data. From these learned distributions, we generate a set of S i.i.d. scenarios, $\boldsymbol{w}_1, \ldots, \boldsymbol{w}_S$, where $\boldsymbol{w}_i := \left[w_i^{\mathrm{T}}(0), \ldots, w_i^{\mathrm{T}}(N-1) \right]^{\mathrm{T}}$, $i = 1, \ldots, S$. Then the chance-constrained problem (11) can be approximated with the following deterministic problem

$$
\begin{aligned}
\min_{\boldsymbol{u}} \quad & \boldsymbol{c}^{\mathrm{T}} \boldsymbol{u} \Delta k \\
\text{s.t.} \quad & \boldsymbol{G}_{xk} x(k) + \boldsymbol{G}_u \boldsymbol{u} \leq \boldsymbol{g} - \max_{i=1,\ldots S} \boldsymbol{G}_w \boldsymbol{w}_i \qquad (12) \\
& \boldsymbol{F} \boldsymbol{u} \leq \boldsymbol{f},
\end{aligned}
$$

where the $\max$ operator is applied element-wise to $\boldsymbol{G}_w \boldsymbol{w}_i$.

Problem (12) can be treated as a multiparametric linear problem by considering the vector $x(k)$ as a vector of parameters [18], [19]. Once the multiparametric problem has been solved off line, assuming that a full measurement of the state $x(k)$ is available at the current time k, the explicit solution is available as a piecewise affine function of $x(k)$

$$
u(k) = Q_i x(k) + q_i \quad \text{if} \quad H_i x(k) \leq K_i \qquad (13)
$$

where the polyhedral sets $\mathcal{X}_i := \{H_i x \leq K_i\}$, with $i = 1 \ldots N$ are a state space partition.

The parameters of the PWA function in (13), Q_i, q_i and the polyhedral sets $\mathcal{X}_i$, can be computing by using the MPT toolbox [30], where a parametric programming solver is implemented.

We remark that the constraints in Problem (12) can be softened to guarantee that the problem with sampled constraints is always feasible.

IV. EXPERIMENTAL RESULTS

A. Experimental Setup

We consider a laboratory room of approximatively 80m^2 in the ground floor of the Q-building of the Royal Institute of Technology (KTH) campus in Stockholm. The room has a concrete heavyweight structure with limited glass surface and one external wall, facing South-East, which is partially shaded by a parking lot. As summarized in Figure 2, its HVAC system is composed of two parts: the ventilation system, supplying fresh air, and a radiator heating system.

Fresh air is supplied by a central balanced ventilation system that operates only between 7:00 and 16:00 during work days. The ventilation system pre-conditions fresh air from outside and distributes it at a temperature of about 20-21°C. Part of this generated air flow is then conveyed directly into the room, while part can be further cooled by a cooling
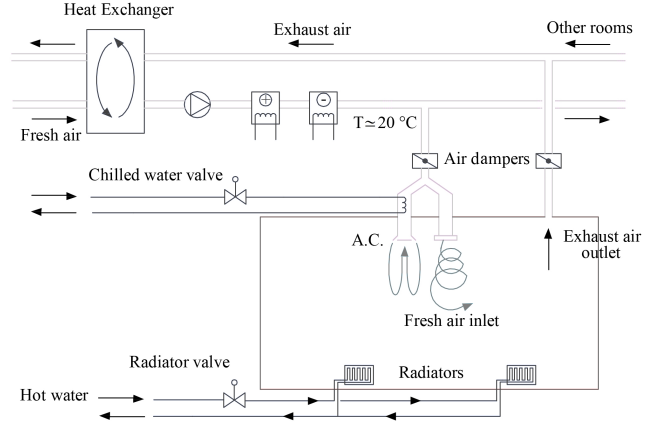
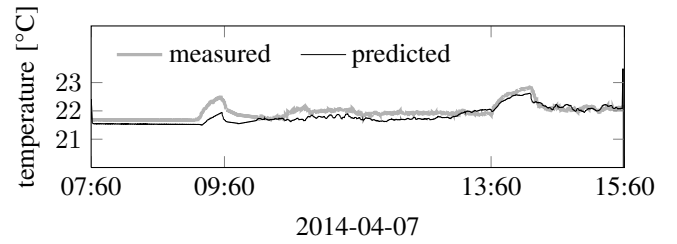

Fig. 2. Scheme of the HVAC system of the testbed.

Fig. 3. Validation of the air temperature model (1a) against measured data.

coil. The controllable actuators of the ventilation system are three: two dampers, that regulate the opening of the inflow and outflow ducts, and a valve, that regulates the temperature of the air chilling circuit. When the central fan is on, a minimum level of the air flow rate is supplied independently of the occupancy level in the room.

The heating system uses instead radiators as final units. The hot water circulating in them is provided by a district heating system and has a temperature that is determined by the external temperature conditions. The unique controllable actuator is the valve regulating the flow of the hot water.

Figure 3 validates the thermal model (1a) by comparing measured and simulated indoor temperature driven by the same disturbances and the same control inputs for a time horizon of 9 hours (which is the prediction horizon considered in the ESMPC problem), starting from the same measured initial condition. The model appears capable of capturing the thermal dynamics in the room with an acceptable accuracy. Indeed, the difference between the measured and the estimated indoor temperature is always smaller than 1 °C. Moreover seven days have been simulated, resulting in similar performance and an average Root-Mean-Square Error of Prediction (RMSEP) equal to 2.19.

The model can be further improved by analyzing the measured and simulated indoor temperature under different weather and occupancy profiles.

B. Evaluation of Experimental Results

We here compare two different controllers *i)* the current practice, a simple control logic with PI control loops and switching logic, indicated by the acronym "AHC" (from Akademiska Hus, the company managing the building of the testbed); *ii)* our ESMPC.

For the latter we consider a prediction horizon of 9 hours and a sampling time of 10 minutes, hence $N = 54$. The optimal control laws are computed with Matlab on an Intel(R) Core(TM)i7-2600 CPU, 3.40 GHz and 8 GB of RAM. It takes 20 minutes on average to compute a optimal control law as a PWA function consisting of 800 polyhedra in the set of admissible initial conditions. After computing all the required state feedback control laws (according the control architecture 1), the profiles of the CO_2 levels, the indoor temperature and the control inputs are obtained by applying Algorithm 1.

We aim at testing the performance of our ESMPC in terms of energy use and comfort as well as the effect of a different number of scenarios in real experiments. The controllers are tested on three consecutive days in March 2014, 18-20, for 9 hours each day, from 7:00 to 16:00. The comfort range of the indoor temperature has been set between 20 °C and 22 °C.

Figure 4 shows then the experimental results. We run the AHC on the 19th, the ESMPC with 10 scenarios on the 18th and the ESMPC with 1000 scenarios on the 20th. Then we compare the AHC with the ESMPC with 10 scenarios in Test 1 and with the ESMPC with 1000 scenarios in Test 2.

Each column of Figure 4 refers to one of the tests above, while each row depicts the disturbances (i.e., outdoor temperature and occupancy), the control inputs (i.e., supply air temperature and massflow) and the controlled indoor temperature and CO_2 levels. The horizontal axis reports the time period of the experiments, from 7:00 to 16:00. Despite the cold season, both controllers in both tests require cooling when there is people in the room. The demand for cooling during the cold season can be explained by the relatively high internal gains, due to occupancy and equipment, and by the limited and moderately insulated external walls surface. This implies that the thermal indoor dynamics are significantly affected by the occupancy patterns.

Notice that on the 18th and on the 20th, the days when the ESMPC was applied, both the outdoor temperature and the occupancy was higher compared to the day when AHC was employed. In particular, the occupancy was significantly higher on the 20th, after lunch time. Despite these more challenging disturbances, the temperature profiles resulting from the ESMPC are very similar to the ones obtained by AHC. The ESMPC is capable to keep the temperature profile close to the upper bound, although the ESMPC with 10 scenarios violated this bound more often.

Despite the temperature profiles resulting form the two controllers being indeed close, the ESMPC achieved this result at a significant lower energy cost. This is a promising performance, especially considering the higher occupancy

and outdoor temperatures that the ESMPC needed to compensate. This can be clearly seen by looking at the required massflow in Figure 4: the AHC turns on/off the cooling system quite often, while the ESMPC uses the cooling system fewer times, i.e., only when really convenient. E.g., consider Test 2 at 14:30, and notice how there is a peak in the massflow that tries to compensate the increased outdoor temperature and the upcoming peak in the occupancy (8 people).
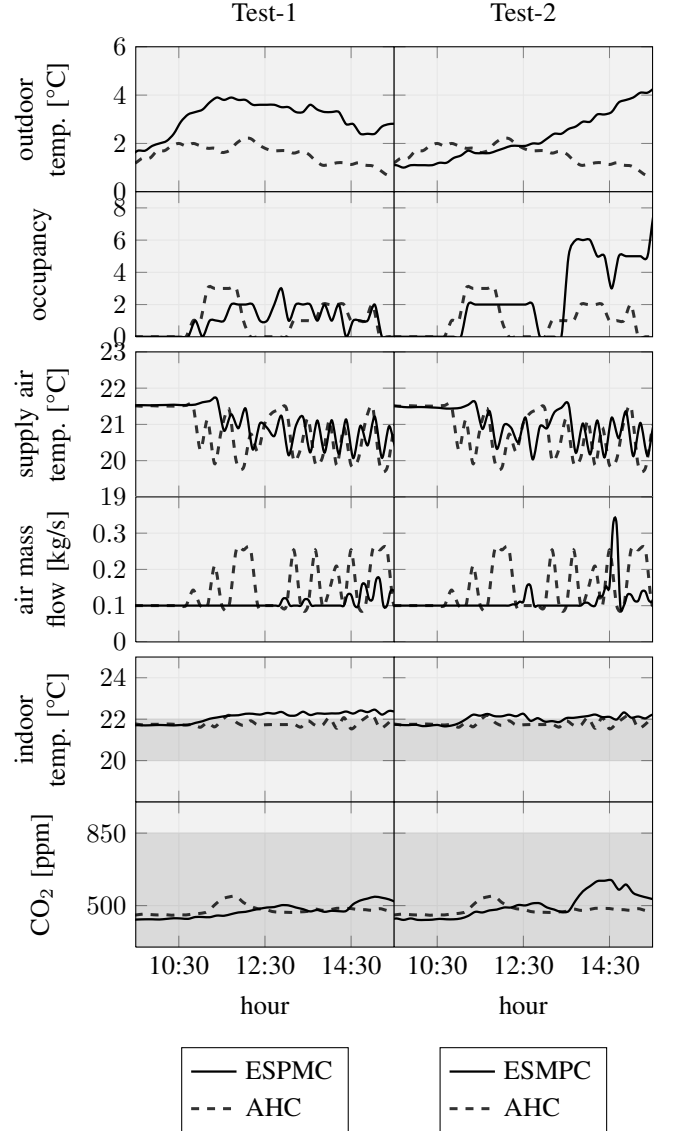

Fig. 4. Disturbances, CO_2 levels, indoor temperatures and control inputs profiles for high- and low-occupancy experimental tests. The shaded areas represent the comfort bounds.

We consider the total energy usage as performance index, calculated as $E_{tot} = c_{pa} \sum_{k=0}^{N-1} \dot{m}_{vent}(k) \big| T_{sa}(k) - T_{room}(k) \big| \Delta k$ [kWh]. Numerically, for the ESMPC $E_{tot} = 0.77$kWh on the 20th, and $E_{tot} = 0.79$kWh on the 18th, while for the AHC $E_{tot} = 1.04$kWh, approximatively $31 - 33\%$ higher.

V. Conclusions and Future Studies

In this work we propose an Explicit Scenario-based Model Predictive Control (ESMPC) controller for Heating, Ventilation and Air Conditioning (HVAC) systems, aiming to diminish the energy required to maintain indoor thermal comfort and acceptable CO_2 levels.

To account for the probabilistic nature of the disturbances affecting the building, we endowed the Model Predictive Control (MPC) scheme with a learning module that infers the statistics of the disturbances from the data without requiring any Gaussian assumption for the statistics of the disturbances. Since implicit MPCs schemes requires solving on-line optimization problems and dedicated hardware, it can be prohibitive to integrate them into a building management system. To cope with these problems we developed explicit versions of the previously implemented strategies, which lead to controllers with simpler hardware and software requirements.

Our explicit approach is, in our opinion, more advanced and complete than the explicit MPC controllers for HVAC schemes currently existing in literature. Indeed, we both consider dynamics of CO_2 *and* temperature, and (at the same time) consider a scenario-based approach, that accounts for the uncertainties on the forecasts of the disturbances.

The strategy has then been implemented and tested on a real room of a university building, showing that the resulting actuation laws can be more effective than the current practice.

The promising results achieved in real experimentations motivate efforts to improve the method, in particular by applying the procedure and the results illustrated in [29], [28]

A further important future work is towards the generalization of the control scheme to the case of whole buildings, which leads to increased modeling effort. Another relevant direction is to investigate how it is possible to adapt the Piecewise Affine (PWA) control laws when the statistics of the disturbances are updated.

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