Regularized Deconvolution-based Approaches for Estimating Room Occupancies

A. Ebadat, G. Bottegal, D. Varagnolo, B. Wahlberg, *Fellow, IEEE,* and K. H. Johansson, *Fellow, IEEE*

***Abstract*—We address the problem of estimating the number of people in a room by means of commonly available information in standard HVAC systems. We propose to use classical system identification concepts: first identify linear dynamics relating occupancy levels, CO_2 concentration levels and room temperature signals, and then formulate the occupancy estimation problem as a deconvolution one. The estimated occupancy is thus the input that best trades off explaining the currently measured CO_2 levels and respecting a regularity condition. This condition corresponds to *fused lasso* priors (i.e., since rooms occupancy levels are assumed piecewise constant, we thus promote piecewise constant solutions by adding an ℓ_1 norm-dependent term to the associated cost function). We propose various estimation algorithms, each one tailored for different levels of available information (namely, the availability of measurements of actuation levels of the venting systems, and of flags signalling when doors open and close). We also provide conditions under which these estimators provide correct estimates within a certain probability. We eventually corroborate the validity of the strategy by means of experiments on real buildings, and numerically assess how the accuracy of the estimates improves with the additional information.**

***Index Terms*—Occupancy estimation, System Identification, Deconvolution, Regularization**

***Note to Practitioners*—Home automation systems benefit from recognizing automatically the human presence in the built environment. But having dedicated sensors is costly, and so it may be preferable to detect occupancies using information that is already existing in standard buildings, so to do not incur in additional costs. Here we show how to estimate occupancy levels starting from measurements of CO_2 temperature and venting levels, and of door openings / closing events, i.e., the pieces of information that is more often available in modern buildings. Our aim is to compare how much these pieces of information are important for occupancy estimation purposes. Technologically speaking, we build algorithms on top of classical *system identification* procedures. Our algorithms nonetheless have a limitation, that is of requiring pilot data, i.e., measuring real occupancy patterns for some time (information needed to train the estimators). Extensions should thus address this limitation, and remove the necessity of training data. Other applications of our algorithms include the estimation of occupancy flows in buildings.**

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I. INTRODUCTION

Estimating occupancy levels in rooms is essential for home automation purposes, e.g., to automate the control of lighting, thermostats, security locks, home entertainment systems, and to improve the energetic performance of Heating, Ventilation and Air Conditioning (HVAC) systems [1], [2], [3]. Estimating occupancy is thus a key enabling factor for improving comfort in smart buildings and energy efficiency.

Direct experience indicates that some standard off-the-shelf dedicated hardware for occupancy estimation (such as cameras and Radio-Frequency Identification (RFID) tags) may be both insufficiently accurate for the employment in HVAC control systems, be inducing large additional deployment and maintenance costs and be associated with installation feasibility problems in old buildings. Moreover, hardware-based occupancy detectors may trigger privacy concerns.

Consequently, an interesting question is whether hardware-based people counters can be replaced by software-based occupancy estimators that employ only that information that is available in standard existing HVAC systems (mainly CO_2 concentrations and temperatures), which information is meaningful to process, and what type of statistical processing leads to efficient estimators.

The main objective of this paper is to study the above questions, and propose occupancy estimators that retrieve information on the number of occupants from commonly available signals, i.e., measurements of CO_2 levels and temperature, HVAC actuation levels (i.e., the amount of fresh air injected in the rooms), and information on door opening / closing events.

Literature review: the strategies addressing the problem of estimating the occupancy levels in rooms and buildings can be categorized into *hardware-based* and *model-based* approaches.

The first category includes methods working with dedicated hardware such as cameras, RFIDs, etc. [4], [5], [6]. As mentioned before, the applicability of these methods are restricted to certain situations due to their potential drawbacks.

In the second category, instead, occupancy levels are inferred indirectly using dynamical models that relate environmental signals with occupancy. These models may be obtained by employing data-driven techniques (i.e., *identification-based* methods) or by exploiting knowledge of the underlying physical laws (i.e., *physics-based* methods). The latter techniques comprise strategies based on mass balance equations or first principles considerations to relate the number of occupiers, CO_2 concentration, temperature and humidity [7], [8], [9], [10]. Instead, identification-based approaches aim at estimating input-output models from data-sets of past measured data.

Successful approaches exploit machine learning techniques such as Support Vector Machines (SVMs), Neural Networks (NNs) and Hidden Markov Models (HMMs) based on CO_2 features (e.g., averages of the signals in time, first / second-order temporal differences) [11], [12].

Statement of contributions: this paper, extension of [13], extends and characterizes a two-tiers hardware-free identification-based estimation strategy.

The first tier of the strategy assumes the availability of both environmental and pilot data, i.e., true occupancy levels, for a short and well-defined period of time. This information is the only one used to model the room under consideration, and no other a-priori knowledge is assumed.

The second tier of the strategy starts formulates the occupancy estimation problem as an inverse problem, i.e., searches the inputs that best explain the measured data given the identified model. In this tier we assume the occupancy signal be *piecewise constant* and integer, and then cast the estimation as a *fused-lasso* problem [14].

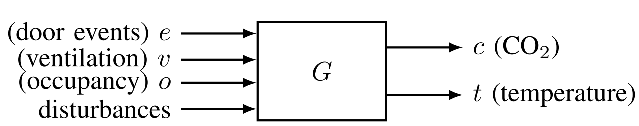
A contribution of the manuscript is to derive different estimators that consider different information sources. More specifically, we consider the case of adding knowledge of HVAC actuation signals (how much air is injected in the room), and the case of adding signals that flagg when doors are opened or closed.

An other contribution is the analysis of the statistical performance of the estimators. We form bounds on the probability of obtaining incorrect estimates, given the levels of measurement noise, the identified model and the design parameters of the estimators.

Structure of the manuscript: Section II formulates the mathematical problem and the solution methodology. Sections III and IV describe respectively how to identify the model of the room from a training set, and how to exploit this model for estimation purposes. Section V characterizes the performance of the estimator from a statistical perspective. Section VI describes how to modify the original estimation strategy when considering also HVAC actuation levels and information on doors openings and closings. Section VII introduces the considered estimation performance indexes, the experimental setup and the results of the estimation processes. Section VIII then wraps some conclusions, remarks, and ideas for future directions. Proofs are collected in the Appendix.

II. Problem definition and methodology

We consider the following schematic representation of the dynamics of the concentration of the CO_2 and temperature in a room under well-mixed air assumptions, i.e., in a room where these quantities are assumed to be spatially constant.



In the above scheme $c(k)$ represents the concentration of CO_2, $t(k)$ the temperature, $v(k)$ the amount of injected fresh air, $o(k)$ the occupancy, all at time k. The variable $e(k)$ is a boolean measurements of door opening and closing events, i.e., flagging if at time k somebody has potentially entered / exited the room. G represents an initially unknown dynamic system relating disturbances, events, ventilation and building occupancy levels with temperature and CO_2 concentration signals.

The first problem we consider is to find an effective algorithm that transforms measurements of $c(k), c(k-1), \ldots$ and $t(k), t(k-1), \ldots$ into estimates of $o(k)$. Our proposal is the following two-tiers estimator:

- **Tier 1, training phase:** from pilot data on $c(k)$, $t(k)$, and $o(k)$, identify a LTI system that captures the dynamics of G (Section III);
- **Tier 2, test phase:** from measurements of $c(k)$ and $t(k)$ and the estimated model of the room estimate $o(k)$ (Section IV).

The first phase addresses a *system identification* problem, while the second phase addresses a *deconvolution* problem.

The second problem we consider is to characterize the proposed estimator in terms of detection error, i.e., probability of obtaining wrong estimates as a function of the parameters of the estimator.

The third problem we consider is how to extend the estimator so to include information on venting levels $v(k), v(k-1), \ldots$, and door openings / closing events $e(k), e(k-1), \ldots$. We shall see that, while including venting levels does not change the structure and main properties of the estimator, accounting for door openings and closings change the problem by adding some opportune constraints.

III. Identification of the room model

In this section we describe how to obtain a model for G starting from pilot data on $c(k)$, $t(k)$, and $o(k)$.

As in [15], [16], [17], [18], [19], we assume the environmental signals to be stationary, the dynamics of the room to be discrete Linear Time Invariant (LTI), measurement devices to be synchronized and operate at the same sample time.

The dynamics of the room can be expressed as

$$\begin{bmatrix} c(k) \\ t(k) \end{bmatrix} = \begin{bmatrix} \boldsymbol{G}_c\left(q^{-1}\right) \\ \boldsymbol{G}_t\left(q^{-1}\right) \end{bmatrix} \begin{bmatrix} c(k-1) \\ t(k-1) \\ o(k-1) \end{bmatrix} + \begin{bmatrix} w_c(k) \\ w_t(k) \end{bmatrix}, \qquad (1)$$

where, without loss of generality,

$$\begin{aligned} \boldsymbol{G}_c\left(q^{-1}\right) &:= \left[G_c^c\left(q^{-1}\right) \quad G_c^t\left(q^{-1}\right) \quad G_c^o\left(q^{-1}\right) \right], \\ \boldsymbol{G}_t\left(q^{-1}\right) &:= \left[G_t^c\left(q^{-1}\right) \quad G_t^t\left(q^{-1}\right) \quad G_t^o\left(q^{-1}\right) \right], \end{aligned}$$

are matrix polynomials with all the entries with the same order. The processes $w_c(k)$, $w_t(k)$ are white Gaussian noises, independent of each other, representing the innovation process, i.e., that part of $c(k)$ and $t(k)$ that cannot be predicted from past measurements.

To estimate the polynomials $\boldsymbol{G}_c\left(q^{-1}\right)$ and $\boldsymbol{G}_t\left(q^{-1}\right)$ that model the system we consider a classical Prediction Error Method (PEM) paradigm, i.e., consider the best linear one-step-ahead predictor of the outputs, namely

$$\begin{bmatrix} \widehat{c}(k|k-1) \\ \widehat{t}(k|k-1) \end{bmatrix} = \begin{bmatrix} \boldsymbol{G}_c\left(q^{-1}\right) \\ \boldsymbol{G}_t\left(q^{-1}\right) \end{bmatrix} \begin{bmatrix} c(k-1) \\ t(k-1) \\ o(k-1) \end{bmatrix}, \qquad (2)$$

obtained by simply neglecting the noise processes. Then, using PEM-based techniques we can obtain $\widehat{\boldsymbol{G}}_c\left(q^{-1}\right)$ and $\widehat{\boldsymbol{G}}_t\left(q^{-1}\right)$, such that the variance of the prediction errors $c(k)-\widehat{c}(k|k-1)$ and $t(k)-\widehat{t}(k|k-1)$ on the data collected during the training phase, is minimized. From (2) it follows that the predictors $\widehat{c}(k|k-1)$ and $\widehat{t}(k|k-1)$ exploit the same information of the past.

In Figure 1 the correlation functions ($\bar{c}(\cdot)$, $\bar{o}(\cdot)$, $\bar{t}(\cdot)$ represent signals stripped of the mean)

$$r_{c,o}(m) := \frac{\sum_{k=0}^{T_{T_s}} \bar{c}(k)\bar{o}(k-m)}{\sqrt{\left(\sum_{k=0}^{T_{T_s}} \bar{c}(k)^2\right)\left(\sum_{k=0}^{T_{T_s}} \bar{o}(k)^2\right)}} ,$$
$$r_{t,o}(m) := \frac{\sum_{k=0}^{T_{T_s}} \bar{t}(k)\bar{o}(k-m)}{\sqrt{\left(\sum_{k=0}^{T_{T_s}} \bar{t}(k)^2\right)\left(\sum_{k=0}^{T_{T_s}} \bar{o}(k)^2\right)}} , \tag{3}$$

are plotted. The correlation functions are computed using the dataset considered throughout the manuscript. It can be promptly seen that the signal mostly correlated with the occupancy is the CO_2 level. For this reason, in the rest of the paper we shall consider only the predictor $\widehat{c}(k|k-1)$ and thus focus on the identification of $\widehat{\boldsymbol{G}}_c\left(q^{-1}\right)$.

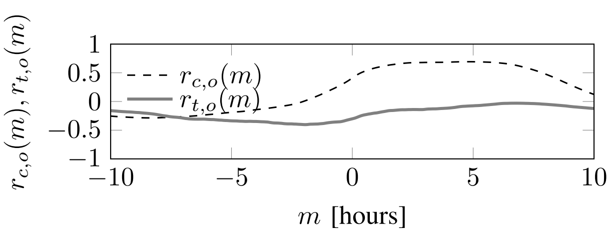


Figure 1. Empirical cross-correlations between occupancy and either temperature ($r_{t,o}(m)$) or CO_2 ($r_{c,o}(m)$), computed using the dataset considered throughout the manuscript (sampling time $t_s = 5$ minutes).

We then consider the nonparametric approach of estimating the system's impulse responses instead of searching for the optimal coefficients of the polynomials $G_c^o\left(q^{-1}\right)$, $G_c^t\left(q^{-1}\right)$ and $G_c^c\left(q^{-1}\right)$. The coefficients of the estimated impulse responses can indeed be truncated to a fixed large number p and then used to form the aforementioned polynomials[1].

Let then $\boldsymbol{g}_c$, $\boldsymbol{g}_t$, and $\boldsymbol{g}_o$ be column vectors containing the impulse responses related to $c(k)$, $t(k)$, and $o(k)$, respectively, and

$$\begin{aligned} \boldsymbol{g} &:= \begin{bmatrix} \boldsymbol{g}_c^T & \boldsymbol{g}_t^T & \boldsymbol{g}_o^T \end{bmatrix}^T , \\ \phi_c(k) &:= \begin{bmatrix} c(k-1) & \dots & c(k-p) \end{bmatrix} , \\ \phi_t(k) &:= \begin{bmatrix} t(k-1) & \dots & t(k-p) \end{bmatrix} , \\ \phi_o(k) &:= \begin{bmatrix} o(k-1) & \dots & o(k-p) \end{bmatrix} , \end{aligned} \tag{4}$$

with $c(k) = t(k) = o(k) = 0$ if $k \leq 0$. Then, instead of estimating $\boldsymbol{g}_c$, $\boldsymbol{g}_t$, and $\boldsymbol{g}_o$ via a classical Least-Squares (LS) approach (technique usually leading to unsatisfactory outcomes due to the high variance of the estimates), the problem is formulated as

$$\widehat{\boldsymbol{g}} = \arg\min_{\{\boldsymbol{g}\in\mathbb{R}^{3p}\}} \|\boldsymbol{c}_{tr} - \Phi\boldsymbol{g}\|_2^2 + \gamma\left(\|\boldsymbol{g}_c\|_P^2 + \|\boldsymbol{g}_t\|_P^2 + \|\boldsymbol{g}_o\|_P^2\right) , \tag{5}$$

[1]In this paper we set $p = 50$.

i.e., as a regularized LS where:

- $\|\boldsymbol{g}\|_P^2 = \boldsymbol{g}^T P \boldsymbol{g}$ with P a positive definite weighting matrix penalizing candidate impulse responses which do not decay to zero for large values of the time index. In this way, P favorites outcomes $\widehat{\boldsymbol{g}}$ that well represent impulse responses of stable systems. Here we set the matrix P as $P = K_\beta^{-1}$, with
$$\left[K_\beta\right]_{i,j} = \beta^{\max\{i,j\}}, \quad 0 < \beta < 1 , \tag{6}$$
i.e., the *stable spline kernel* [20], [21], [22], depending on the hyperparameter β, whose optimal choice is discussed below;
- γ is a positive real number representing a trade-off between variance and bias of the estimator, leading for $\gamma = 0$ to the LS estimate of $\boldsymbol{g}$ (solution usually with a high variance).

The optimal values of γ and β can be computed via either cross validation-based strategies [23] or empirical Bayes techniques [20], [21]. Once these values have been established, the solution can be computed in closed form as in the parametric case [24] as

$$\widehat{\boldsymbol{g}} = \left(\Phi^T\Phi + \gamma D_P\right)^{-1} \Phi^T \boldsymbol{c}_{tr} , \tag{7}$$

where D_P is block diagonal with four blocks all equal to P.

IV. Deconvolution of the occupancy levels

We now build an estimator $\widehat{o}(k)$ of $o(k)$ as a function of the measurements $c(k)$ and $t(k)$ and estimated room dynamics $\widehat{G}_c^c$, $\widehat{G}_c^t$, $\widehat{G}_c^o$. Let then

$$\widehat{c}(k|k-1) = \widehat{\boldsymbol{G}}_c\left(q^{-1}\right)\begin{bmatrix} c(k-1) \\ t(k-1) \\ o(k-1) \end{bmatrix} , \tag{8}$$

and consider the CO_2 levels prediction error

$$\varepsilon(k) := c(k) - \widehat{c}(k|k-1). \tag{9}$$

Under the stated assumptions $\varepsilon(k)$ is a homoscedastic zero-mean Gaussian white noise [25]. Substituting (8) into (9) and rearranging properly,

$$\begin{aligned} &\widehat{G}_c^o\left(q^{-1}\right) o(k-1) \\ &= c(k) - \left[\widehat{G}_c^c\left(q^{-1}\right)\ \widehat{G}_c^t\left(q^{-1}\right)\right]\begin{bmatrix} c(k-1) \\ t(k-1) \end{bmatrix} - \varepsilon(k), \end{aligned} \tag{10}$$

where the unknowns are only $o(k-1)$ and $\varepsilon(k)$, since

$$\widetilde{c}(k) := c(k) - \left[\widehat{G}_c^c\left(q^{-1}\right)\ \widehat{G}_c^t\left(q^{-1}\right)\right]\begin{bmatrix} c(k-1) \\ t(k-1) \end{bmatrix}$$

can be computed given the available information. Thus (10) becomes

$$\widetilde{c}(k) = \widehat{G}_c^o\left(q^{-1}\right) o(k-1) + \varepsilon(k), \tag{11}$$

which shows that the problem of estimating the unknown $o(\cdot)$ is a deconvolution problem, i.e., the unknown occupancy signal $\widehat{o}(k)$ is the input that best describes the observed output $\widetilde{c}(k)$, given the knowledge of the transfer function $\widehat{G}_c^o$. Since $\varepsilon(k)$ is assumed white and Gaussian, the natural approach to this problem would be to employ a LS estimator of $o(\cdot)$, since this would minimize the overall variance of the estimation error [23, Chap. 7]. More specifically, let $\widehat{G}_c^o\left(q^{-1}\right) = g_1 q^{-1} + \ldots + g_p q^{-p}$ and let the test set be indexed by the time instants $0, \ldots, T_{ts}$. Considering the auxiliary notation

$$\widehat{G} := \begin{bmatrix} g_1 & 0 & \ldots & & & 0 \\ g_2 & g_1 & & & & \\ \vdots & \ddots & \ddots & & & \vdots \\ g_p & \cdots & g_2 & g_1 & & \\ & \ddots & & \ddots & \ddots & \\ 0 & & g_p & \cdots & g_2 & g_1 \end{bmatrix} \qquad \boldsymbol{o} := \begin{bmatrix} o(0) \\ \vdots \\ o(T_{ts}-1) \end{bmatrix} \qquad \widetilde{\boldsymbol{c}} := \begin{bmatrix} \widetilde{c}(1) \\ \vdots \\ \widetilde{c}(T_{ts}) \end{bmatrix}$$

the LS estimator can be formulated as the optimization problem

$$\widehat{\boldsymbol{o}} = \arg\min_{\widetilde{\boldsymbol{o}} \in \mathbb{R}_+^{T_{ts}}} \left\| \widetilde{\boldsymbol{c}} - \widehat{G}\, \widetilde{\boldsymbol{o}} \right\|_2^2, \tag{12}$$

which performance are usually unsatisfactory, since the high variance of the estimates make the solution not reflect suitable room occupancy patterns. To overcome this issue we then account for the prior information $o(k)$ *is non-negative, integer, and piecewise constant*.

We thus formulate the deconvolution problem as finding that least-changing positive piecewise constant input signal that can reasonably reduces the mismatch between the estimated and measured outputs of the system.

More precisely, let

$$\boldsymbol{o} := \begin{bmatrix} o(k-N) \\ \vdots \\ o(k-1) \end{bmatrix}, \quad \widetilde{\boldsymbol{c}} := \begin{bmatrix} \widetilde{c}(k-N+1) \\ \vdots \\ \widetilde{c}(k) \end{bmatrix}$$

$$\Delta o(i) := o(i) - o(i-1), \quad \Delta \boldsymbol{o} := \left[\Delta o(1), \ldots, \Delta o(N-1)\right].$$

The estimation problem then becomes

$$\widehat{\boldsymbol{o}}(k-1) = \arg\min_{\widetilde{\boldsymbol{o}} \in \mathbb{N}_+^N} \left\| \widetilde{\boldsymbol{c}} - \widehat{G}\, \widetilde{\boldsymbol{o}} \right\|_2^2 + \lambda \left\| \Delta \widetilde{\boldsymbol{o}} \right\|_0, \tag{13}$$

where:

- $\widehat{\boldsymbol{o}}(k-1)$ is a N-dimensional vector with the estimated values of occupancy at the time instants $k-1, \ldots, k-N$ (for online estimation purposes one might consider to use just its first entry $\widehat{o}(k-1)$);
- the first summand on the RHS represents the LS estimator of the occupancy, that tries to match the estimated and measured outputs of the system;
- $\left\|\cdot\right\|_0$, the ℓ_0 norm, counts the number of variations of the candidate inputs, thus penalizing non-piecewise constant candidate inputs;
- λ is a regularization parameter that trades off the two previous terms and that is discussed in details in Section IV-A.

Unfortunately, Problem (13) is a non-convex non-linear integer program, and cannot be solved efficiently. To circumvent this computational drawback we propose two relaxations: first, substitute the ℓ_0 norm with the ℓ_1-norm [26, Sec. 3.4], which represents its best convex relaxation. Second, extend the domain of the plausible inputs to $\mathbb{R}_+^N$ instead of $\mathbb{N}_+^N$, so that the estimation problem becomes

$$\widehat{\boldsymbol{o}}(k-1) = \left\lfloor \arg\min_{\widetilde{\boldsymbol{o}} \in \mathbb{R}_+^N} \left\| \widetilde{\boldsymbol{c}} - \widehat{G}\, \widetilde{\boldsymbol{o}} \right\|_2^2 + \lambda \left\| \Delta \widetilde{\boldsymbol{o}} \right\|_1 \right\rceil, \tag{14}$$

with $\lfloor \cdot \rceil$ the vector-wise rounding operator. Problem (14) is a particular case of *fused-lasso* estimator, where the solution is searched among *sparse* regressor vectors where small changes are favored with respect to big ones, and the strength of this preference is dictated by the regularization parameter λ.

The parameter N plays an important role in (14), since it defines the amount of data employed for estimating $\widehat{\boldsymbol{o}}(k-1)$ (and in particular $\widehat{o}(k-1)$) at each time instant. Clearly, a large value of N yields more accurate estimates, since more information is used. However, a large value of N brings computational issues which could make the computation of (14) too slow for online operations. Thus, as discussed in Section VII-B, a good choice of N should consider both these aspects.

A. Finding the optimal regularization parameter λ

The regularization parameter λ establishes the typical variability of the room occupancy signal. Indeed, large values of λ penalize changes in the value of estimated occupancy, leading to estimates that are constant for long periods of times. Small values of λ, instead, lead to occupancy signals with high frequency components, thus behaving similarly to the outcomes of the LS estimator (which is obtained by setting $\lambda = 0$).

A reasonable choice of λ is given by that value $\widehat{\lambda}$ that gives the best estimation performance during the training phase. This optimal value can then be computed by the following algorithm:

1) define a grid Λ of candidate values of λ;
2) for each $\lambda \in \Lambda$ solve Problem (14) using the $c(k)$, $t(k)$ and $v(k)$ collected *during the training phase*, obtaining $\widehat{\boldsymbol{o}}(\lambda)$, i.e., an occupancy estimate as function of λ;
3) compute the optimal regularization parameter as

$$\widehat{\lambda} = \arg\min_{\lambda \in \Lambda} \left\| \widehat{\boldsymbol{o}}(\lambda) - \boldsymbol{o} \right\|_2^2, \tag{15}$$

with $\boldsymbol{o}$ the occupancy levels measured during the training phase.

V. Characterization of the occupancy estimator

We now derive relations between the probability of obtaining wrong occupancy estimates and the quantities parameterizing the estimator, namely the identified linear models, the noisiness levels of the measurements, and the regularization parameter λ.

Our first result regards the performance of the estimator when the occupancy is constant in a window of N past values.

Proposition 1 Let $\sigma_{\boldsymbol{\varepsilon}}$ be the variance of the noise in (11), N the window length in the estimator, and λ the regularization parameter. Assume that $o(k)$ is a constant signal. Define

$$\Delta := \begin{bmatrix} -1 & 1 & & \\ & \ddots & \ddots & \\ & & -1 & 1 \end{bmatrix} \in \mathbb{R}^{N-1 \times N} \tag{16}$$

and $V^T := \left(\Delta \widehat{G}^{-1}\right)^{\dagger}$, where $(X)^{\dagger}$ denotes the Moore-Penrose pseudoinverse of X. Then $\widehat{o}(k)$ is detected as constant with probability of at least α if

$$\lambda^2 > \sigma_{\boldsymbol{\varepsilon}}^2 \chi_{\alpha}^{-1}(N) \|V_m\|^2 \tag{17}$$

where $\chi_{\alpha}^{-1}(N)$ is the inverse of the chi-square Cumulative Distribution Function (CDF) with N degrees of freedom for the corresponding probability α and $\|V_m\|^2 := \max_i \|V_i\|^2$, with V_i the i-th row of V.

The following result studies the case where $o(k)$ has a variation.

Proposition 2 Let $\sigma_{\boldsymbol{\varepsilon}}$ be the variance of the noise in (11), N the window length in the estimator, λ the regularization parameter. Define $\bar{\Delta} \in \mathbb{R}^{N-1 \times N-1}$, obtained removing the first column of Δ and $\bar{V}^T := \left(\bar{\Delta} \bar{H}^{-1}\right)^{-1}$. Assume that the first value of the estimated occupancy is set to the true one, i.e., $\widehat{o}(k-N) = o(k-N)$, and that $o(k)$ has a unique discontinuity given by a variation of one unit. Then, $\widehat{o}(k)$ is detected as constant, i.e., there is a missed change with probability of at least α if

$$\lambda^2 > \sigma_{\boldsymbol{\varepsilon}}^2 \chi_{\alpha}^{-1}(N) \|\bar{V}_1\|^2 + (1 + o(k-N)^2) \|\bar{V}_1\|^4 , \tag{18}$$

where $\bar{V}_1$ is the first row of $\bar{V}$.

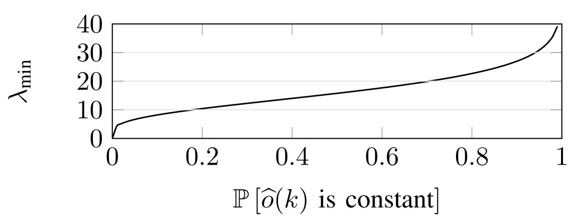


Figure 2. Graphical representation of bound (17) as functions of the probability α for a given $\sigma_{\boldsymbol{\varepsilon}}^2$.

The previous results can easily be extended to the more general case where the true occupancy is piecewise constant with ρ discontinuities of +1 units. The sufficient condition to estimate a constant signal in this case will be

$$\left\| \frac{\boldsymbol{\varepsilon}}{\sigma_{\boldsymbol{\varepsilon}}} \right\|^2 < \frac{\lambda^2 - (\rho + o(k-N)^2) \left\|\bar{V}_1\right\|^4}{\sigma_{\boldsymbol{\varepsilon}}^2 \left\|\bar{V}_1\right\|^2}$$

or, equivalently, $\lambda^2 > \sigma_{\boldsymbol{\varepsilon}}^2 \chi_{\alpha}^{-1}(N) \|V_1\|^2 + (\rho + o(k - N)^2) \|V_1\|^4$.

VI. Accounting for additional information

We here address the cases where the available information contains the additional signals $v(k)$ (venting levels, i.e., how much fresh air is injected in the room) and $d(k)$ (door events, i.e., if the door has been opened / closed at time k).

A. Accounting for venting levels

When the signal $v(k)$ is available, immediate generalizations lead to express (8) as

$$\widehat{c}(k|k-1) = \widehat{\boldsymbol{G}}_c\left(q^{-1}\right) \begin{bmatrix} c(k-1) \\ t(k-1) \\ v(k-1) \\ o(k-1) \end{bmatrix}, \tag{19}$$

$$\widehat{\boldsymbol{G}}_c\left(q^{-1}\right) = \left[\widehat{G}_c^c\left(q^{-1}\right) \;\; \widehat{G}_c^t\left(q^{-1}\right) \;\; \widehat{G}_c^v\left(q^{-1}\right) \;\; \widehat{G}_c^o\left(q^{-1}\right)\right].$$

This leads to natural extensions of the system identification procedures of Section III, with $\boldsymbol{g} := \left[\boldsymbol{g}_c^T \;\; \boldsymbol{g}_t^T \;\; \boldsymbol{g}_o^T\right]^T$, and

$$\widehat{\boldsymbol{g}} = \arg \min_{\{\boldsymbol{g} \in \mathbb{R}^{3p}\}} \|\boldsymbol{c}_{tr} - \Phi \boldsymbol{g}\|_2^2 + \gamma\left(\|\boldsymbol{g}_c\|_P^2 + \|\boldsymbol{g}_t\|_P^2 + \|\boldsymbol{g}_o\|_P^2\right).$$

The same extension applies to the deconvolution step: (11) indeed remains structurally the same as soon as $\widetilde{c}(k)$ is redefined as

$$\widetilde{c}(k) := c(k) - \left[\widehat{G}_c^c\left(q^{-1}\right) \;\; \widehat{G}_c^t\left(q^{-1}\right) \;\; \widehat{G}_c^v\left(q^{-1}\right)\right] \begin{bmatrix} c(k-1) \\ t(k-1) \\ v(k-1) \end{bmatrix}.$$

B. Accounting for door opening and closing events

Assume now the knowledge of $e(k)$, i.e., of flags stating if at time k somebody has potentially entered / exited the room. More precisely, assume that $e(k) = 0 \Rightarrow o(k) = o(k-1)$, while no information is added on $o(k)$ when $e(k) \neq 0$ is available.

As for the system identification problem, information on $e(k)$ is non-influential, i.e., does not modify the derivations performed in Section III, since during the identification the knowledge on the occupancy levels is considered complete. In other words, $o(k)$ contains already the information in $e(k)$.

As for the deconvolution problem, knowing $e(k)$ changes the structure of the estimator, since $e(k)$ naturally constraints the estimand occupancy levels to be identical when $e(k) = 0$. More precisely, knowing $e(k)$ corresponds to know the sparsity pattern of the to-be-reconstructed signal. This imply that the regularization term $\|\Delta \widetilde{o}\|_0$ in (13) is a constant factor that does not depend on the decision variables, and thus that (13) is equivalent to the Integer Quadratic Program (IQP)

$$\begin{aligned} \widehat{\boldsymbol{o}}(k-1) = \arg \min_{\widetilde{\boldsymbol{o}} \in \mathbb{N}_+^N} \quad & \left\|\widetilde{\boldsymbol{c}} - \widehat{G}\, \widetilde{\boldsymbol{o}}\right\|_2^2 \\ \text{s.t.} \quad & \Delta \widetilde{o}(k) = 0 \text{ for all } e(k) \neq 0. \end{aligned} \tag{20}$$

Following the motivations that brought from (13) to (14), (20) can thus be relaxed with

$$\begin{aligned} \widehat{\boldsymbol{o}} = \arg \min_{\widetilde{\boldsymbol{o}} \in \mathbb{R}_+^{T_{ts}}} \quad & \left\lfloor \left\|\widetilde{\boldsymbol{c}} - \widehat{G}\, \widetilde{\boldsymbol{o}}\right\|_2^2 \right\rfloor \\ \text{s.t.} \quad & \Delta \widetilde{o}(k) = 0 \text{ for all } e(k) \neq 0. \end{aligned} \tag{21}$$

Due to the lack of the regularization term, (21) does not require tunings of regularization parameters, as did instead for (14).

For sake of completeness, we then further modify (21) and add back the ℓ_1 regularization term $\lambda \|\Delta \widetilde{o}\|_1$ to obtain

$$\widehat{\boldsymbol{o}} = \arg \min_{\widetilde{\boldsymbol{o}} \in \mathbb{R}_+^{T_{ts}}} \left\lfloor \left\| \widetilde{\boldsymbol{c}} - \widehat{G}\, \widetilde{\boldsymbol{o}} \right\|_2^2 + \lambda \|\Delta \widetilde{\boldsymbol{o}}\|_1 \right\rfloor \quad \text{s.t.} \quad \Delta \widetilde{o}(k) = 0 \text{ for all } e(k) \neq 0. \tag{22}$$

As noticed before, this regularization term corresponds thus to favor, in the occupancy signal, small changes to big ones, with the strength of this preference dictated by the regularization parameter λ. Obviously, implementing estimator (22) requires to find the optimal λ, as described in Section IV-A.

VII. Experiments

Tests have been performed in one of the rooms of the KTH ACL-HVAC testbed, see http://hvac.ee.kth.se/ for more information. The information collected, available at http://hvac.ee.kth.se/datasets.html, comprises two weeks of measurements of CO_2 and temperature levels from HDH sensors, and of venting, cooling, and heating actuation levels from the central HVAC system. Occupancy levels were manually registered for the whole period, with a time accuracy of 1 minute. To uniform the sampling times of the various signals (5 minutes), or in case of missing measurements, the information was resampled using linear interpolation schemes. The first week was used as a training set, while the second week was used as a test set.

A. Definition of the performance indexes

We consider four performance indexes: *i)* the *Mean Squared Error (MSE)* (23), characterizing the relative estimation errors; *ii)* the *accuracy* (25), reporting how many times the estimator returns the correct value; *iii)* the *false positive / false negative occupancy detection rates* (28), describing the ability of discriminating the presence / absence of occupants in terms of false positives (when the room is estimated to be occupied while it is not) and false negatives (when the room is estimated to be empty while it is not).

The MSE associated with $\boldsymbol{o}$ and $\widehat{\boldsymbol{o}}$ is

$$\text{MSE}\left(\widehat{\boldsymbol{o}}\right) := \frac{\|\widehat{\boldsymbol{o}} - \boldsymbol{o}\|_2^2}{\|\boldsymbol{o}\|_2^2}. \tag{23}$$

To define the other performance indexes we then transform the signals $\boldsymbol{o}$, $\widehat{\boldsymbol{o}}$ with codomain $\mathbb{N}_+$ (number of occupants) to signals with codomain $\{0, 1\}$ (room is non occupied, room is occupied) through indicator functions, i.e., through

$$\mathbb{1}\left(o(k)\right) := \begin{cases} 1 & \text{if } o(k) > 0 \\ 0 & \text{otherwise} \end{cases}, \quad \mathbb{1}\left(\boldsymbol{o}\right) := \begin{bmatrix} \mathbb{1}\left(o(1)\right) \\ \vdots \\ \mathbb{1}\left(o(N)\right) \end{bmatrix}. \tag{24}$$

Given (24), the accuracy of the estimate $\widehat{\boldsymbol{o}}$ is

$$\text{Acc}\left(\widehat{\boldsymbol{o}}\right) := \frac{N - \sum_{k=1}^{N} \mathbb{1}\left(o(k) - \widehat{o}(k)\right)}{N}. \tag{25}$$

To define the false positive / negative rates we introduce

$$\mathcal{N}_\theta := \left\{ t \text{ s.t. } \mathbb{1}\left(o(k)\right) = \theta \right\}, \tag{26}$$

dividing the time indexes in two sets: $\mathcal{N}_0$, for the k's for which the room was not occupied, and $\mathcal{N}_1$, for the k's for which the room was occupied. With this it is possible to capture the mistakes "the room is estimated to be occupied while it is empty", "the room is considered empty while it is occupied" with

$$\widehat{\beta}(\theta) := \frac{1}{|\mathcal{N}_\theta|} \sum_{k \in \mathcal{N}_\theta} \mathbb{1}\left(\widehat{o}(k)\right), \tag{27}$$

where we remark that the summation is performed over the set $\mathcal{N}_\theta$. With (27) the false positive and false negative rates become

$$\text{FP}\left(\widehat{\boldsymbol{o}}\right) := \widehat{\beta}(0), \qquad \text{FN}\left(\widehat{\boldsymbol{o}}\right) := 1 - \widehat{\beta}(1). \tag{28}$$

B. Summary of the results

1) Evaluation of the importance of additional information: Let us postpone discussing the choice of the parameters λ and N to the following subsections, and assume for now that these parameters have been already optimally chosen. Table I then numerically assesses the value of knowing the ventilation levels $v(k)$ and the door openings / closing flags $e(k)$, while Figure 3 depicts graphically the realizations of the results.

Estimator	MSE	Accuracy	FP	FN
(14)	0.208	0.822	0.039	0.028
(14)+v	0.124	0.888	0.007	0.018
(21)	0.217	0.884	0.001	0.028
(22)	0.109	0.886	0.006	0.008

Table I
Comparison of the performance of estimators (14), (14) with knowledge of ventilation levels $v(k)$, (21) and (22).

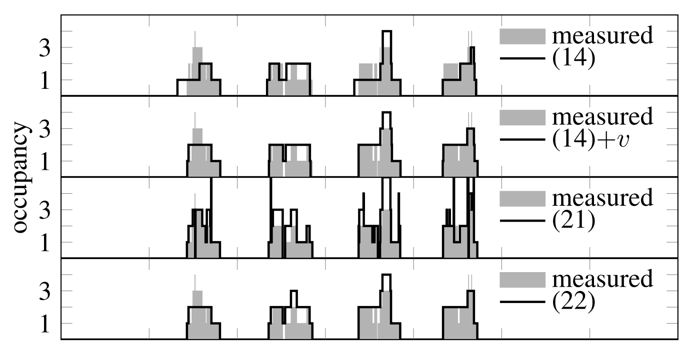


Figure 3. Realizations of the estimates for the test set considered in our experiments for the various estimators proposed in this manuscript.

2) Evaluation of the sensitivity to the regularization parameter λ: Let us postpone discussing the choice of the parameter N to the following subsection, and assume for now that this parameter has been already optimally chosen. λ dictates the typical variability of the estimated occupancy patterns, and is set during the test phase as that value $\widehat{\lambda}$ that leads to the best estimation performance *in the training set*. Since the best value for the test set may be different from the best value in the training set, it is important to evaluate the effects of this unavoidable mismatch.

Figure 4 plots the MSE for different λ's for estimator (14)+v for both the training and test sets. The dependency on λ appears relatively weak in the test set, and the MSE of the training and test sets attain their minima at approximately the same point. This suggests that the proposed estimation strategy for λ is reliable and effective.

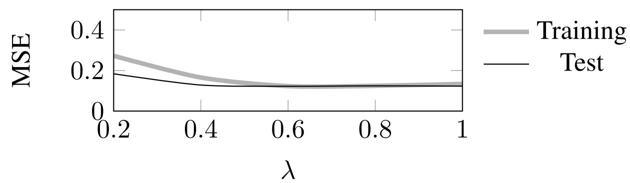


Figure 4. Sensitivity of the performance of estimator (14)+v w.r.t. the choice of λ.

3) Evaluation of the sensitivity to the optimization horizon N: N trades-off computational requirements with information: the larger the optimization horizon, the more information the estimators have about the dynamics of the system. Intuition suggests that, after a certain horizon length, adding more information does not improve the estimation performance, i.e., after this horizon the old dynamics do not influence the current estimates. The results shown in Figure 5 indicate that this length is, in our experiments, of about 5 days.

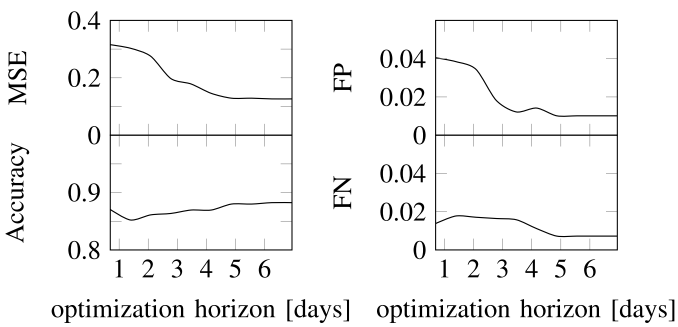


Figure 5. Dependency of the performance of estimator (22) w.r.t. the choice of N.

VIII. Conclusions

We proposed methods for estimating occupancy levels in closed environments that exploit different amounts of information, and aimed at understanding which is the most meaningful information to estimate how occupancy levels change in time. The main assumption made is that the estimator can, for learning purposes and for a short period of time, access to direct measurements of the true occupancy levels.

All the proposed estimation strategies first obtain Linear Time Invariant (LTI) models by suitable identification techniques, and then formulate the occupancy estimation problem as a regularized deconvolution problem (where the regularization exploits prior information on the features of the searched signal). The obtained results show that adding information on ventilation and door opening / closing events can double the performance indexes of the estimators.

We also analyzed the theoretical statistical performance of the estimators, and shown that the probability of obtaining wrong estimates can be suitably bounded once one knows the measurements noise variance.

The idea considered in this paper can be extended towards the construction of occupancy estimators for whole buildings, and thus for the identification of building occupancy pattern models. Moreover, since the dynamics are assumed linear, it may be possible to adapt the models identified in a single room to other rooms of the same building, by an opportune rescaling of the identified impulse responses accounting variations in the structural properties of rooms.

Another appealing idea is to exploit blind system identification techniques to estimate both the system dynamics and the building occupancy at the same time, thus removing the assumption on the availability of the building occupancy signal for a given period.

Appendix

A. Proof of Proposition 1

The proof is divided in 3 main parts: *i)* rewrite (14), derive the dual of the new problem and the structure of its solution. *ii)* find some analytical relations between the estimated and the true occupancy levels. *iii)* exploit these relations to derive bounds that characterize the statistical performance of the estimator.

i): Introduce the variable $\boldsymbol{z} := \Delta\widetilde{\boldsymbol{o}}$ and rewrite (14) as

$$\arg \min_{\substack{\widetilde{\boldsymbol{o}} \in \mathbb{R}^N_+ \\ \boldsymbol{z} \in \mathbb{R}^{N-1}}} \frac{1}{2}\left\|\widetilde{\boldsymbol{c}} - \widehat{G}\widetilde{\boldsymbol{o}}\right\|_2^2 + \lambda\left\|\boldsymbol{z}\right\|_1 \quad \text{s.t.} \quad \boldsymbol{z} = \Delta\widetilde{\boldsymbol{o}}, \tag{29}$$

where, for the purposes of the proof, the function $\lfloor\cdot\rceil$ (the vector-wise rounding operator) is omitted. The Lagrangian of (29) is then

$$\mathcal{L}\left(\widetilde{\boldsymbol{o}}, \boldsymbol{z}, \boldsymbol{u}\right) = \frac{1}{2}\left\|\widetilde{\boldsymbol{c}} - \widehat{G}\,\widetilde{\boldsymbol{o}}\right\|_2^2 + \lambda\left\|\boldsymbol{z}\right\|_1 + \boldsymbol{u}^T\left(\Delta\widetilde{\boldsymbol{o}} - \boldsymbol{z}\right) \tag{30}$$

where $\boldsymbol{u}$ is the Lagrange multiplier. The dual problem, obtained minimizing $\mathcal{L}$ w.r.t. $\widetilde{\boldsymbol{o}}$ and $\boldsymbol{z}$, is [27]

$$\arg\min_{\boldsymbol{u}\in\mathbb{R}^N} \frac{1}{2}\left\|\widetilde{\boldsymbol{c}} - \left(\Delta\widehat{G}^{-1}\right)^T\boldsymbol{u}\right\|_2^2 \quad \text{s.t.} \quad \left|\boldsymbol{u}\right|_\infty \leq \lambda. \tag{31}$$

We notice that, since $\widehat{G}$ is a lower triangular matrix, $\widehat{G}$ admits inverse as soon as $g_1 \neq 0$. This is then satisfied as soon as there is (only) one delay in the effects of the occupancy on the CO_2 levels of the room.

To obtain the structure of the dual solution, consider again the derivative of the Lagrangian with respect to $\boldsymbol{z}$

$$\begin{aligned}\min_{\boldsymbol{z}}\mathcal{L}\left(\widetilde{\boldsymbol{o}}, \boldsymbol{z}, \boldsymbol{u}\right) &= \min_{\boldsymbol{z}}(\lambda\left\|\boldsymbol{z}\right\|_1 - \boldsymbol{u}^T\boldsymbol{z}) \\ &= \begin{cases} 0 & \text{if } \left|\boldsymbol{u}\right|_\infty \leq \lambda, \\ -\infty & \text{otherwise} \end{cases}.\end{aligned} \tag{32}$$

Let then $\widehat{\boldsymbol{u}}_\lambda$ be the dual solution and $\widehat{\boldsymbol{z}} = \Delta\widehat{\boldsymbol{o}}_\lambda$ be the primal solution of (29) for a specific λ. Given the computations above, it satisfies

$$\widehat{u}_{\lambda,i} \in \begin{cases} \{+\lambda\} & \text{if} \quad (\Delta\widehat{\boldsymbol{o}})_i > 0, \\ \{-\lambda\} & \text{if} \quad (\Delta\widehat{\boldsymbol{o}})_i < 0, \\ [-\lambda,\ \lambda] & \text{if} \quad (\Delta\widehat{\boldsymbol{o}})_i = 0. \end{cases} \tag{33}$$

In other words, in order to maximize (32), the i^{th} element of the dual solution, i.e., $\widehat{u}_{\lambda,i}$, should be $+\lambda$ if the corresponding element in the primal solution is positive and it should be $-\lambda$ if the corresponding element in the primal solution is negative, see [27]. For those elements of the primal solution with zero values we can only say that the dual problem must satisfy the condition $|\boldsymbol{u}|_\infty \leq \lambda$.

From (33), one can conclude that $|\widehat{u}_{\lambda,i}| \neq \lambda$, only if $(\Delta\widehat{\boldsymbol{o}})_i = 0$.

ii): relax problem (31) by removing the ∞-norm constraint. The resulting problem is a unconstrained Least-Squares (LS) problem, with solution

$$\boldsymbol{u}_{\text{LS}} = \left(\Delta\widehat{G}^{-1}\right)^{T^\dagger} \widetilde{\boldsymbol{c}}. \tag{34}$$

If $\|\boldsymbol{u}_{\text{LS}}\|_\infty < \lambda$ holds, then two facts hold:

1) $\boldsymbol{u}_{\text{LS}}$ is also the solution of problem (31);
2) due to the last implication described in *i)*, $\Delta\widehat{\boldsymbol{o}} = 0$, i.e., the estimated occupancy is a constant signal.

These two facts connect variations in the estimate $\Delta\widehat{\boldsymbol{o}}$ with the measured signal $\widetilde{c}$, considering $V^T := \left(\Delta\widehat{G}^{-1}\right)^\dagger$, since they read as

$$\|V\widetilde{\boldsymbol{c}}\|_\infty < \lambda \quad \Rightarrow \quad \Delta\widehat{\boldsymbol{o}} = 0. \tag{35}$$

To explicit $\widetilde{\boldsymbol{c}}$, consider that the vectorized version of (11) reads as

$$\widetilde{\boldsymbol{c}} = \widehat{G}\boldsymbol{o} + \boldsymbol{\varepsilon}, \tag{36}$$

with $\boldsymbol{\varepsilon} \in \mathbb{R}^{T_{ts}}$ white and Gaussian innovation, and $\boldsymbol{o}$ the true occupancy signal. Rewriting V as

$$V = \left(\Delta\widehat{G}^{-1}\widehat{G}^{-T}\Delta^T\right)^{-1}\Delta\widehat{G}^{-1} \tag{37}$$

and eventually, substituting (37) into (35), we rewrite the latter as

$$\left\|\left(\Delta\widehat{G}^{-1}\widehat{G}^{-T}\Delta^T\right)^{-1}\Delta\widehat{G}^{-1}\left(\widehat{G}\boldsymbol{o} + \boldsymbol{\varepsilon}\right)\right\|_\infty < \lambda\,, \tag{38}$$

which in turn implies $\Delta\widehat{\boldsymbol{o}} = 0$. As can be seen, (38) relates conditions on the true occupancy $\boldsymbol{o}$ and the innovation process $\boldsymbol{\varepsilon}$ with conditions on the final estimate $\widehat{\boldsymbol{o}}$.

iii): we now analyze the case when the true occupancy is constant ($\Delta\boldsymbol{o} = \boldsymbol{0}$). In this case condition (38) reads as

$$\|V\boldsymbol{\varepsilon}\|_\infty < \lambda \quad \Rightarrow \quad \Delta\widehat{\boldsymbol{o}} = 0, \tag{39}$$

that is equivalent to

$$\left\{|\langle V_i, \boldsymbol{\varepsilon}\rangle|^2 < \lambda^2\right\}_{i=1,\ldots,N} \quad \Rightarrow \quad \Delta\widehat{\boldsymbol{o}} = 0\,. \tag{40}$$

The Cauchy-Schwarz inequality yields $\langle V_i, \boldsymbol{\varepsilon}\rangle|^2 \leq \|V_i\|^2\|\boldsymbol{\varepsilon}\|^2$. Letting $\|V_m\|^2 := \max_i \|V_i\|^2$, the sufficient condition for (39) becomes

$$\|V_m\|^2\|\boldsymbol{\varepsilon}\|^2 < \lambda^2 \quad \Rightarrow \quad \Delta\widehat{\boldsymbol{o}} = 0. \tag{41}$$

In (41) V_m is known, while $\boldsymbol{\varepsilon}$ is white Gaussian noise: thanks to the Prediction Error Method (PEM) paradigm, $\boldsymbol{\varepsilon}_i \sim \mathcal{N}(0, \sigma^2_{\boldsymbol{\varepsilon}})$, with $\sigma^2_{\boldsymbol{\varepsilon}}$ estimated during the system identification phase. It thus follows that

$$\left\|\frac{\boldsymbol{\varepsilon}}{\sigma_{\boldsymbol{\varepsilon}}}\right\|^2 = \sum_{i=1}^{T_{ts}}\left(\frac{\boldsymbol{\varepsilon}_i}{\sigma_{\boldsymbol{\varepsilon}}}\right)^2 \sim \chi^2(N)\,, \tag{42}$$

where $\chi^2(N)$ is a Chi-squared distribution with N degrees of freedom. Thus, with the probability of at least α, $\|\boldsymbol{\varepsilon}\|^2$ will have the following upper bound

$$\|\boldsymbol{\varepsilon}\|^2 \leq (\sigma_{\boldsymbol{\varepsilon}})^2\,\chi^{-1}_\alpha(N)\,, \tag{43}$$

where $\chi^{-1}_\alpha(N)$ is the inverse of the chi-square cdf with N degrees of freedom for the corresponding probability α. Substituting (43) into (41), we get the statement of the proposition.

B. Proof of Proposition 2

In this case, we impose another constraint on the optimization problem (14) by setting the first element in the occupancy signal to its true value. Using the same approach as in the proof of the proposition 1, we will have (29) subject to $\widetilde{\boldsymbol{o}}(1) = o(k-N)$, where $o(k-N)$ is the true value of the occupancy signal at time $k-N$. Substituting the new constraint $\widetilde{\boldsymbol{o}}(1) = o(k-N)$ into the cost function, one can rewrite (29) as

$$\begin{aligned} \arg \min_{\substack{\bar{\boldsymbol{o}} \in \mathbb{R}^N_+ \\ \boldsymbol{z} \in \mathbb{R}^{N-1}}} \quad & \frac{1}{2}\left\|\bar{\boldsymbol{c}} - \bar{H}\bar{\boldsymbol{o}}\right\|^2_2 + \lambda\left\|\boldsymbol{z} - \bar{\boldsymbol{o}}^*\right\|_1 \\ \text{s.t.} \quad & \boldsymbol{z} = \bar{\Delta}\bar{\boldsymbol{o}}\,, \end{aligned} \tag{44}$$

where

$$\bar{\boldsymbol{o}}^* := \left[o(k-N)\ \ 0\ \ \cdots\ \ 0\right]^T \in \mathbb{R}^{N-1\times N-1}.$$

Using the same approach as before the dual problem for (44) will be

$$\begin{aligned} \arg\min_{\boldsymbol{u}\in\mathbb{R}^N} \quad & \frac{1}{2}\left\|\bar{\boldsymbol{c}} - \left(\bar{\Delta}\bar{H}^{-1}\right)^T\boldsymbol{u}\right\|^2_2 \\ \text{s.t.} \quad & |\boldsymbol{u}|_\infty \leq \lambda, \end{aligned} \tag{45}$$

where the Lagrange multipliers satisfy

$$\widehat{u}_{\lambda,i} \in \begin{cases} \{+\lambda\} & \text{if} \quad \left(\bar{\Delta}\bar{\boldsymbol{o}} - \bar{\boldsymbol{o}}^*\right)_i > 0, \\ \{-\lambda\} & \text{if} \quad \left(\bar{\Delta}\bar{\boldsymbol{o}} - \bar{\boldsymbol{o}}^*\right)_i < 0, \\ [-\lambda,\ \lambda] & \text{if} \quad \left(\bar{\Delta}\bar{\boldsymbol{o}} - \bar{\boldsymbol{o}}^*\right)_i = 0. \end{cases} \tag{46}$$

Notice that $\bar{\Delta}$ is invertible and thus the condition (38) for this case reads as

$$\left\|\left(\bar{V}\bar{V}^T\right)^1 o^* + \left(\bar{V}\bar{V}^T\right)^k \pm \bar{V}\boldsymbol{\varepsilon}\right\|_\infty < \lambda \quad \Rightarrow \quad \bar{\Delta}\bar{\boldsymbol{o}} - \bar{\boldsymbol{o}}^* = 0 \tag{47}$$

where $\left(\bar{V}\bar{V}^T\right)^k$ is the k-th column of $\bar{V}\bar{V}^T$ and $\bar{V} = (\bar{H}\bar{\Delta}^{-1})^T$. Notice that this is a upper triangular Toeplitz matrix, satisfying (letting $\bar{V}_j$ be the j-th row of $\bar{V}$)

$$\left\|\bar{V}_1\right\|^2 \geq \left\|\bar{V}_2\right\|^2 \geq \ldots \geq \left\|\bar{V}_N\right\|^2. \tag{48}$$

This implication refers to the case where the estimator makes the error of not finding the change in the occupancy signal at time k.

The ∞-norm above can be expanded, as before, to obtain the component-wise equivalent condition

$$\left\{ \left| \langle \bar{V}_i, \bar{V}_1 \rangle o^* + \langle \bar{V}_i, \bar{V}_k \rangle \pm \langle \bar{V}_i, \boldsymbol{\varepsilon} \rangle \right| < \lambda \right\}_{i=1,\dots,N} \Rightarrow \bar{\Delta}\bar{\boldsymbol{o}} = \bar{\boldsymbol{o}}^* \tag{49}$$

or, using the bilinearity of inner products,

$$\left\{ \left| \langle \bar{V}_i, \bar{V}_1 o(k-N) + \bar{V}_k \pm \boldsymbol{\varepsilon} \rangle \right| < \lambda \right\}_{i=1,\dots,N} \quad \Rightarrow \quad \bar{\Delta}\bar{\boldsymbol{o}} = \bar{\boldsymbol{o}}^*. \tag{50}$$

Cascading now Cauchy-Schwarz and triangular inequalities with (48) and (50) it is possible to derive the sufficient condition

$$\begin{aligned} &\left\| \bar{V}_1 \right\|^2 \left(\left\| \bar{V}_1 \right\|^2 o(k-N)^2 + \left\| \bar{V}_1 \right\|^2 + \left\| \varepsilon \right\|^2 \right) < \lambda^2 \\ &\Rightarrow \quad \bar{\Delta}\bar{\boldsymbol{o}} = \bar{\boldsymbol{o}}^* \end{aligned} \tag{51}$$

or, equivalently,

$$\left\| \frac{\boldsymbol{\varepsilon}}{\sigma_{\boldsymbol{\varepsilon}}} \right\|^2 < \frac{\lambda^2 - \left(1 + o(k-N)^2\right) \left\| \bar{V}_1 \right\|^4}{\sigma_{\boldsymbol{\varepsilon}}^2 \left\| \bar{V}_1 \right\|^2} \quad \Rightarrow \quad \bar{\Delta}\bar{\boldsymbol{o}} = \bar{\boldsymbol{o}}^*. \tag{52}$$

Same considerations as in the previous case thus follow and (52) can be read as

$$\lambda^2 > \sigma_{\boldsymbol{\varepsilon}}^2 \chi_\alpha^{-1}(N) \|\bar{V}_1\|^2 + (1 + o(k-N)^2) \|\bar{V}_1\|^4. \tag{53}$$

References

[1] F. Oldewurtel, D. Sturzenegger, and M. Morari, "Importance of occupancy information for building climate control," *Applied Energy*, vol. 101, pp. 521–532, Jan. 2013.

[2] O. Guerra Santin, L. Itard, and H. Visscher, "The effect of occupancy and building characteristics on energy use for space and water heating in Dutch residential stock," *Energy and Buildings*, vol. 41, no. 11, pp. 1223–1232, Nov. 2009.

[3] T. A. Nguyen and M. Aiello, "Energy intelligent buildings based on user activity: A survey," *Energy & Buildings*, vol. 56, pp. 244–257, 2013.

[4] N. Li, G. Calis, and B. Becerik-Gerber, "Measuring and monitoring occupancy with an RFID based system for demand-driven HVAC operations," *Automation in Construction*, vol. 24, pp. 89–99, July 2012.

[5] H. Wang, Q.-S. Jia, Y. Lei, Q. Zhao, and X. Guan, "Estimation of occupant distribution by detecting the entrance and leaving events of zones in building," in *2012 IEEE International Conference on Multisensor Fusion and Integration for Intelligent Systems*. Ieee, Sept. 2012, pp. 27–32.

[6] S. Meyn, A. Surana, Y. Lin, S. M. Oggianu, S. Narayanan, and T. A. Frewen, "A Sensor-Utility-Network Method for Estimation of Occupancy in Buildings," in *IEEE Conference on Decision and Control*, 2009, pp. 1494–1500.

[7] T. Leephakpreeda, R. Thitipatanapong, T. Grittiyachot, and V. Yungchareon, "Occupancy-Based Control of Indoor Air Ventilation: A Theoretical and Experimental Study," *Science Asia*, vol. 27, no. 4, pp. 279–284, 2001.

[8] F. Scotton, L. Huang, S. A. Ahmadi, and B. Wahlberg, "Physics-based Modeling and Identification for HVAC Systems," in *European Control Conference*, 2013.

[9] S. Wang, J. Burnett, and H. Chong, "Experimental Validation of CO2-Based Occupancy Detection for Demand-Controlled Ventilation," *Indoor and Built Environment*, vol. 8, no. 6, pp. 377–391, Nov. 1999.

[10] S. Kar and P. K. Varshney, "Accurate estimation of indoor occupancy using gas sensors," in *International Conference on Intelligent Sensors, Sensor Networks and Information Processing*. Ieee, Dec. 2009, pp. 355–360.

[11] K. P. Lam, M. Höynck, B. Dong, B. Andrews, Y.-s. Chiou, D. Benitez, and J. Choi, "Occupancy detection through an extensive environmental sensor network in an open-plan office building," in *IBPSA Conference*, 2009, pp. 1452–1459.

[12] B. Dong, B. Andrews, K. P. Lam, M. Höynck, R. Zhang, Y.-S. Chiou, and D. Benitez, "An information technology enabled sustainability test-bed (ITEST) for occupancy detection through an environmental sensing network," *Energy and Buildings*, vol. 42, no. 7, pp. 1038–1046, July 2010.

[13] A. Ebadat, G. Bottegal, D. Varagnolo, B. Wahlberg, and K. H. Johansson, "Estimation of building occupancy levels through environmental signals deconvolution," in *ACM Workshop On Embedded Systems For Energy-Efficient Buildings*, 2013.

[14] R. Tibshirani and M. Saunders, "Sparsity and smoothness via the fused lasso," *Journal of Royal Statistical Society: Series B (Statistical Methodology)*, vol. 67, no. 1, pp. 91–108, 2005.

[15] G. Mustafaraj, J. Chen, and G. Lowry, "Development of room temperature and relative humidity linear parametric models for an open office using BMS data," *Energy and Buildings*, vol. 42, no. 3, pp. 348–356, 2010.

[16] D. Loveday and C. Craggs, "Stochastic modelling of temperatures for a full-scale occupied building zone subject to natural random influences," *Applied Energy*, vol. 45, no. 4, pp. 295–312, 1993.

[17] G. Lowry and M.-W. Lee, "Modelling the passive thermal response of a building using sparse BMS data," *Applied Energy*, vol. 78, no. 1, pp. 53–62, 2004.

[18] J. C.-M. Yiu and S. Wang, "Multiple ARMAX modeling scheme for forecasting air conditioning system performance," *Energy Conversion and Management*, vol. 48, pp. 2276–2285, 2007.

[19] S. Wu and J.-Q. Sun, "A physics-based linear parametric model of room temperature in office buildings," *Building and Environment*, vol. 50, pp. 1–9, Apr. 2012.

[20] G. Pillonetto, A. Chiuso, and G. De Nicolao, "Prediction error identification of linear systems: a nonparametric Gaussian regression approach," *Automatica*, vol. 47, no. 2, pp. 291–305, 2011.

[21] G. Pillonetto and G. De Nicolao, "A new kernel-based approach for linear system identification," *Automatica*, vol. 46, no. 1, pp. 81–93, 2010.

[22] G. Bottegal and G. Pillonetto, "Regularized spectrum estimation using stable spline kernels," *Automatica*, vol. 49, no. 11, pp. 3199–3209, Nov. 2013.

[23] L. Ljung, *System identification - Theory for the user*, 2nd ed. Prentice-Hall, 1999.

[24] T. Chen, H. Ohlsson, and L. Ljung, "On the estimation of transfer functions, regularizations and Gaussian processes - Revisited," *Automatica*, vol. 48, no. 8, pp. 1525–1535, Aug. 2012.

[25] B. D. Anderson and J. B. Moore, *Optimal Filtering*. Prentice-Hall, 1979.

[26] T. Hastie, R. Tibshirani, and J. Friedman, *The Elements of Statistical Learning: Data Mining, Inference, and Prediction*, 2nd ed. New York: Springer, 2001.

[27] R. J. Tibshirani, *The Solution Path of the Generalized Lasso*. Stanford University, 2011.