3) Fast Dynamics ($\epsilon \to 0$).

$$\mathbb{1}^T\dot{\mathbf{y}}(t) = \mathbb{1}^T\dot{\mathbf{v}}(t) \Longrightarrow y_{ave}(t) = v_{ave}(t) + y_{ave}(0) - v_{ave}(0), \quad (y_{ave} = \tfrac{1}{N}\mathbb{1}^T\mathbf{y})$$
$$\mathbb{1}^T\dot{\mathbf{z}}(t) = \mathbb{1}^T\dot{\mathbf{w}}(t) \Longrightarrow z_{ave}(t) = z_{ave}(t) + w_{ave}(0) - w_{ave}(0), \quad \textit{true } \forall\epsilon$$

$$\begin{cases} \mathbf{v}(t) \to \mathbf{g}(\mathbf{x}(t)) & \text{fast dynamics} \\ \mathbf{w}(t) \to \mathbf{h}(\mathbf{x}(t)) & \\ \dot{\mathbf{y}}(t) \to \left(\frac{1}{N}\mathbb{1}^T\mathbf{g}(\mathbf{x}(t))\right)\mathbb{1} \quad \text{if } y_{ave}(0) = v_{ave}(0) & \\ \dot{\mathbf{z}}(t) \to \left(\frac{1}{N}\mathbb{1}^T\mathbf{h}(\mathbf{x}(t))\right)\mathbb{1} \quad \text{if } z_{ave}(0) = w_{ave}(0) & \\ \hline \dot{\mathbf{x}}(t) = -\mathbf{x}(t) + \dfrac{\frac{1}{N}\mathbb{1}^T\mathbf{g}(\mathbf{x}(t))}{\frac{1}{N}\mathbb{1}^T\mathbf{h}(\mathbf{x}(t))}\mathbb{1}, \Longrightarrow \mathbf{x}(t) \to x_{ave}(t)\mathbb{1} & \text{slow dyn.} \end{cases}$$

If ε is sufficiently small ...

- first subsystem is much faster than second one
- first subsystem is globally exponentially stable