Robustness properties

Proposition

Assume that $f_i \in \mathcal{C}^2$, f_i strictly convex, $x^* \neq \pm\infty$, and

$$\begin{aligned} ||\mathbf{x}(0) - x^*\mathbb{1}|| &= \rho \\ |\mathbb{1}^T(\mathbf{v}(0) - \mathbf{y}(0))| &= \alpha \\ |\mathbb{1}^T(\mathbf{v}(0) - \mathbf{y}(0))| &= \beta \end{aligned}$$

There is a positive scalars $\bar{\varepsilon}, \bar{\rho}, \bar{\alpha}, \bar{\beta}$ and $\phi(\alpha, \beta) : \mathbb{R}^2 \to \mathbb{R} \in \mathcal{C}^0$, s.t. if $\varepsilon < \bar{\varepsilon}, \rho < \bar{\rho}, \alpha < \bar{\alpha}, \beta < \bar{\beta}$ then

$$\lim_{k \to +\infty} \mathbf{x}(k) = \phi(\alpha, \beta)\mathbb{1}, \quad \text{exponentially}$$

and $\phi(0, 0) = x^*$